Three 13th-century views of quantified modal logic

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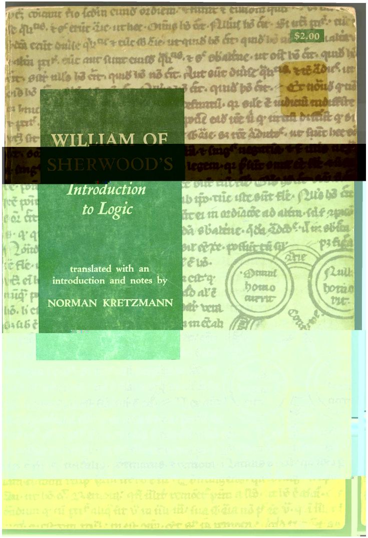
Two reasons to study medieval logic

- To see how similar medieval logic is to modern logic
- To see how different medieval logic is from modern logic



The sources

- William of Sherwood's *Introductiones in logicam*, Latin edition with German translation [11], English translation with commentary [10]).
- *De propositionibus modalibus* [9], author uncertain but probably St Thomas Aquinas.
- Pseudo-Aquinas's *Summa totius logicae Aristotelis* [8].

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- William of Sherwood's *Introductiones in logicam*, Latin edition with German translation [11], English translation with commentary [10]). Written between 1240 and 1248, while Sherwood was a master in the Arts Faculty at the University of Paris.
- *De propositionibus modalibus* [9], author uncertain but probably St Thomas Aquinas. If genuine, it is a juvenile and early work.
- Pseudo-Aquinas's *Summa totius logicae Aristotelis* [8]. Long thought to be by Aquinas, but probably in fact not genuine.

Modes and modal propositions (1)

Definition (Categorical proposition)

A *categorical proposition* is *cuius substantia consistit ex subiecto et praedicato* [Sherwood].

Definition (Mode)

Aquinas *determinatio adiacens rei, quae quidem fit per adiectionem nominis adiectivi, quod determinat substantivum. . . vel per adverbium, quod determinat verbum*—both adverbs and adjectives.

Pseudo-Aquinas *adjacens rei determinatio; idest, determinatio facta per adjectivum*—just adjectives.

Sherwood *determinatio alicuius actus, et secundum hoc convenit omni adverbio*—just adverbs.

Modes and modal propositions (2)

Definition (Modal proposition)

A *modal proposition* is a categorical proposition

(a) to which a mode has been added,

(b) where

- Pseudo-Aquinas: the mode *determinat compositionem ipsam praedicati ad subiectum*
- Sherwood: the mode *determinatio praedicati in subiecto*
- Aquinas: *inhaerentia praedicati ad subjectum modificatur*

The modes: *verum*, *falsum*, *necessarium, impossibile, possibile,* and *contingens*.

Two ways a mode can determine composition

The three authors make the distinction in slightly different ways and with different labels:

Aquinas: *de dicto* vs. *de re*.

Pseudo-Aquinas: *de dicto* vs. *de re*.

Sherwood: nominal modes vs. adverbial modes.

> *Modalis de dicto est, in qua totum dictum subiicitur et modus praedicatur, ut Socrates currere est possibile; modalis de re est, in qua modus interponitur dicto, ut Socratem possibile est currere [Aquinas].*

Quantity & quality

Both categorical and modal propositions have quantity and quality. These determine the inferential relations that hold between pairs or sets of categorical or modal propositions.

The quantity is either singular, particular, universal, or indefinite. The quality is either positive or negative.

Quantity of categorical propositions

A categorical proposition is

- singular when the subject term picks out only one object, because it is a proper name or it is modified by a definitie article such as *hoc* or *illud*.
- particular when the subject term picks out more than one object, because it is modified by a particular quantifier such as *quoddam* or *aliquid*.
- universal when the subject term picks out all objects of which the term can be truly predicated, because is modified by a universal quantifier such as *omnem* or *nullum*.
- indefinite when the subject term refers to some object or objects, but no particular object or objects, because no quantifier or definite article is used, and the subject is not a proper name.

Quantity of modal propositions

Modal *de re* statements have the same quantity as their underlying categorical sentences.

Aquinas and Pseudo-Aquinas: modal *de dicto* statements always have singular quantity, even though they may contain universal or particular quantifiers within them.

Sherwood: when a categorical statement with a nominal mode is interpreted as if is adverbial, then the quantity is determined by the quantity of the underlying categorical. When not interpreted this way, the *dictum* of the sentence is the subject, and this is singular.

Quality of categorical and modal propositions

Quality of a proposition is determined by the presence or absence of a negation:

- for categorical sentences, it is the negation of the composition between the subject and the predicate
- for modal sentences it is the negation of the mode.

If the composition or the mode is affirmed, then the sentence is affirmative, and if it is denied, then it is negative.

> *Propositio modalis dicitur affirmativa vel negativa secundum affirmationem vel negationem modi, et non dicti [Aquinas].*

Inferential relations

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The conversions:

- conversion *per accidens*
- conversion *simplex*
- equivalences which can be generated through the square of opposition.

These implications and conversions are used to develop a modal syllogistic.

Implications (1)

Modes can be combined with negation in one of the following four ways:

- A without negation
- B with more than one negation
- C with one negation, before the mode
- D with one negation, after the mode

Four modes $\times$ four combinations of modes and negations $=$ sixteen syntactically different modes, which can occur in *de re* or *de dicto* propositions.

Implications (2)

Each of the sixteen can be placed into one of four *ordines*:

ordo 1	*possibile* *contingens* *non impossible* *non necessarium non*	*non possibile* *non contingens* *impossibile* *necessarium non*	*ordo* 3
ordo 2	*possibile non* *contingens non* *non impossibile non* *non necessarium*	*non possibile non* *non contingens non* *impossibile non* *necessarium*	*ordo* 4

Figure: The four *ordines*

Omnes propositiones quae sunt in eodem ordine, aequipollent sunt [Aquinas].

Modal square of opposition

The four orders make up the corners of a square of opposition illustrating the inferential relationships:

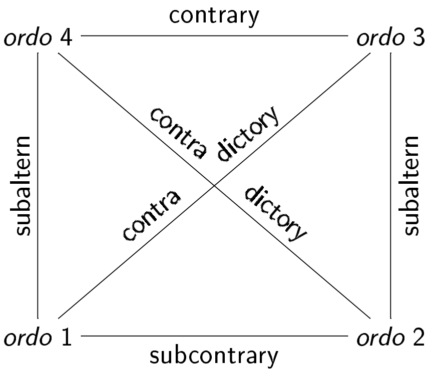

Figure: Modal square of opposition

Conversions

- Simple conversion: exchanges the subject and predicate terms, does not change the quality or the quantity
- Accidental conversion: exchanges the subject and predicate terms, changes quantity from universal to particular or vice versa.

Propositiones de necessario et impossibili eodem modo convertuntur sicut propositiones de inesse, et per idem principium probantur [Pseudo-Aquinas].

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This only applies to modal *de dicto* propositions/propositions with nominal modes, and only to necessary and impossible propositions.

Modal syllogisms

Pseudo-Aquinas discusses modal syllogisms in tract. 7, caps. 13–15.

Modal syllogistic covers syllogisms with different combinations of necessary, impossible, contingent, and assertoric premises. Each combination is considered; if it is valid, no argument is given, if it is invalid, a counterexample is given.

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Problems with the theory:

- No general rules are given (though some specific ones are)
- Pseudo-Aquinas moves between *de dicto* and *de re* formulations indiscriminately.

The rules

In assertoric syllogisms, there are the two general rules, the *dici de omni* and the *dici de nullo*: These rules are often used to argue for the invalidity of certain syllogisms with one modal and one assertoric premise.

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The modal rule: For syllogisms which have one necessary premise and one contingent or possible premise:

> *si aliquod subjectum sit essentialiter sub aliquo praedicato, quicquid contingit sub subjecto, contingit sub praedicato.*

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Example:

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But necessary conclusion does not follow from an assertoric major and a necessary minor.
Example:

Omnis homo est albus.
Omne risibile necessario est homo.
Ergo omne risibile necessario est album.

These are *de re*.

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- The nature of modality
- The truth conditions for modal propostions

The nature of modality

Modern view:

- **Propositional modal logic**: modality is *de dicto* modality. Not clear that *de re* modality can be interpreted in a propositional context in a coherent fashion.
- **Quantified modal logic**: *de re* modality indicates some type of first-order logic. But there is some temptation to say that *de re* statements aren't *really* about modality; they're just about a (perhaps special) type of predicates which we could call, e.g., possibly-P.

The nature of modality

Modern view:

- **Propositional modal logic**:
- **Quantified modal logic**:

Medieval view:

- **Strictly speaking**: Only categorical sentences where the mode determines the inherence of the subject and predicate are really modal. Under this interpretation:

  > *Si enim dicam 'Socratem currere est contingens', idem est secundum rem ac si dicerem 'Socrates contingenter currit' [Sherwood].*

Modifications of inherence

Can modifications of the inherence of a subject in a predicate be represented in first-order modal logic?

If the underlying categorical proposition is universal or particular, then:

- nominal: $\Box\forall x F(x)$
- adverbial: $\forall x\Box F(x)$

Note: This doesn't work for singular or indefinite statements, which have no quantifier.

Two ways to read $\Diamond P(c)$

Given

$$\mathcal{M} = \langle W, R, D, I, V \rangle$$

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We have two ways to read $w \vDash \Diamond P(c)$:

1. There is a world x such that wRx and $\mathcal{M}, x \models_V Py$ where $V(y) = I(c, x)$.
2. There is a world x such that wRx and $\mathcal{M}, x \models_V Py$ where $V(y) = I(c, w)$.

Extending the analysis to complex propositions

Formalise *Omnis homo est possibile currere* as

$$\forall y(Hy \to \Diamond_{dr} Cy)$$

On the *de re* analysis

$$\mathcal{M}, w \models \forall y(Hy \to \Diamond_{dr} Cy)$$

is true if and only if for arbitrary m

if $I(m) \in I(H, w)$, then $\exists x, wRx$ and $\mathcal{M}, x \models_v C(y)$ where $y \in I(m, w)$

The truth conditions of modal sentences

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- Emphasis is placed on truth conditions of propositions considered in isolation. In Kripke semantics, this manifests itself in the choice of R or a restriction on V.

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Medieval view:

- Emphasis is placed on inferential relations between modal propositions:
  - the Square of Opposition,
  - conversions and of modal propositions,
  - classes of valid syllogisms.

Medieval comments on truth conditions

- Pseudo-Aquinas: no explicit truth conditions for modal propositions *considered in themselves*.
  This is surprising considering the stated goal of his treatise:

  Omnes homines natura scire desiderant.
- Aquinas has two sentences on the subject:
- This interpretation corresponds to Sherwood's definition of necessity and impossibility *per se*:

Medieval comments on truth conditions

- Pseudo-Aquinas: no explicit truth conditions for modal propositions *considered in themselves*.
- Aquinas has two sentences on the subject:

  > *Attendendum est autem quod necessarium habet similitudinem cum signo universali affirmativo, quia quod necesse est esse, semper est; impossibile cum signo universali negativo, quia quod est impossibile esse, nunquam est. Contingens vero et possibile similitudinem habent cum signo particulari: quia quod est contingens et possibile, quandoque est, quandoque non est*

- This interpretation corresponds to Sherwood's definition of necessity and impossibility *per se*:

Sherwood & statistical modality (1)

Two ways to use *impossibile* and *necessarium*:

> *uno modo, quod non potest nec poterit nec potuit esse verum, et est impossible per se. . . alio modo, quod non potest nec poterit esse verum, potuit tamen . . . et est impossibile per accidens. Et similiter dicitur necessarium per se, quod non potest nec potuit nec poterit esse falsum. . . Necessarium autem per accidens est, quod non potest nec poterit esse falsum, potuit tamen*

$$
\begin{aligned}
\Box_{ps}\varphi &:= \varphi \wedge G\varphi \wedge H\varphi \\
\Box_{pa}\varphi &:= \varphi \wedge G\varphi \wedge \Diamond\neg H\varphi
\end{aligned}
$$

Figure: Sherwood's necessity operators

Sherwood & statistical modality (2)

Two ways to use *possibile* and *contingens*:

- used of statements which can both be true and be false, and so are neither impossible or necessary = "contingent".
- used of statements which can be true, even if they cannot be false = "possible" under the assumption of $\Box\varphi \rightarrow \Diamond\varphi$

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Per se possibility can be formalized:

$$\Diamond_{ps}\varphi := (\varphi \vee F\varphi \vee P\varphi) \wedge (\neg\varphi \vee F\neg\varphi \vee P\neg\varphi) \tag{1}$$

Interpretation of $\Diamond$

What does $\Diamond$ stand for in

$$\Box_{pa}\varphi := \varphi \wedge G\varphi \wedge \Diamond\neg H\varphi? \tag{2}$$

Not:

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Let w be an arbitrary point where $\Box_{pa}\varphi$ is true. We know then that $w \models \varphi$ and $w \models G\varphi$, and

$$w \models (\neg H\varphi \vee F\neg H\varphi \vee P\neg H\varphi) \wedge (H\varphi \vee FH\varphi \vee PH\varphi) \tag{3}$$

Sherwood's counterfactual truth conditions for necessity *per accidens* cannot involve temporal possibility.

Solution to the problem

Soluton: formally distinguish between *de re* and *de dicto* readings. This exploits the fact that φ is not a simple proposition but is a subject-predicate sentence.

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Example: If *Socrates est necessario currere* is interpreted with necessity *per accidens*, it can be rewritten as

$$C(s) \wedge \Box_{ps} FC(s) \wedge \Diamond_{dr} \neg C(s) \tag{4}$$

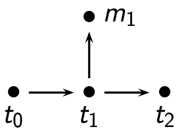

Figure: $\Box_{ps}$ is evaluated w.r.t. t_n, $\Diamond_{dr}$ w.r.t m_1

Concluding remarks

Fitting and Mendelsohn say that

> *for much of the latter half of the twentieth century, there has been considerable antipathy toward the development of modal logic in certain quarters. Many of the philosophical objectors find their inspiration in the work of W.V.O. Quine, who as early as (Quine, 1943), expressed doubts about the coherence of the project... Quine does not believe that quantified modal logic can be done coherently... [4, p. 89]*

This suspicion of quantified modal logic is deep-seated and pervasive among contemporary philosophical logicians.

We have demonstrated that quantified modal logic does not have to be a scary, intractable field of study, but in fact can be developed in a systematic fashion from the logic of simple categorical statements.

Thank You

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