A Tableau for RoBCTL*

John McCabe-Dansted

University of Western Australia

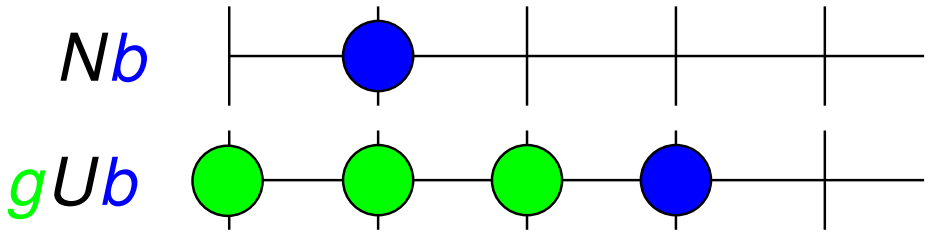
September 8, 2008

RoCTL*

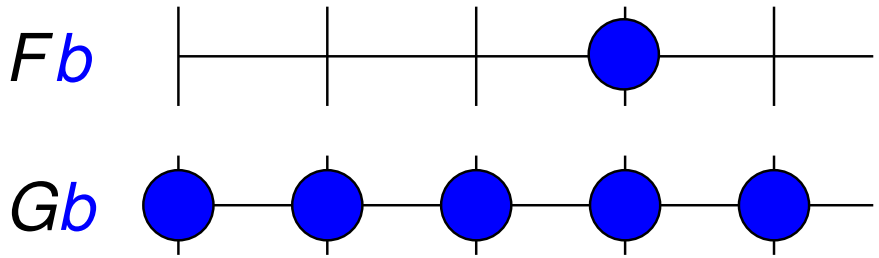
- We proposed a new logic, RoCTL* (McCabe-Dansted et al., 2007), for modelling and verifying specifications that deal with
  - Time (extends CTL*)
  - Obligation (what the system *should* do)
  - Robustness (what the system will do even if *n* failures occur, etc.)
- We motivated its use, apply to simple problems.
- We showed that RoCTL* is decidable
- We now present a tableau based decision procedure
  - For a "bundled" variant RoBCTL*
    - Extends tableau for BCTL* (Reynolds, 2007)

Linear Temporal Logic (LTL)

Classical logic has two base operators $\wedge$, $\neg$
LTL has two additional base operators, Next and Until:

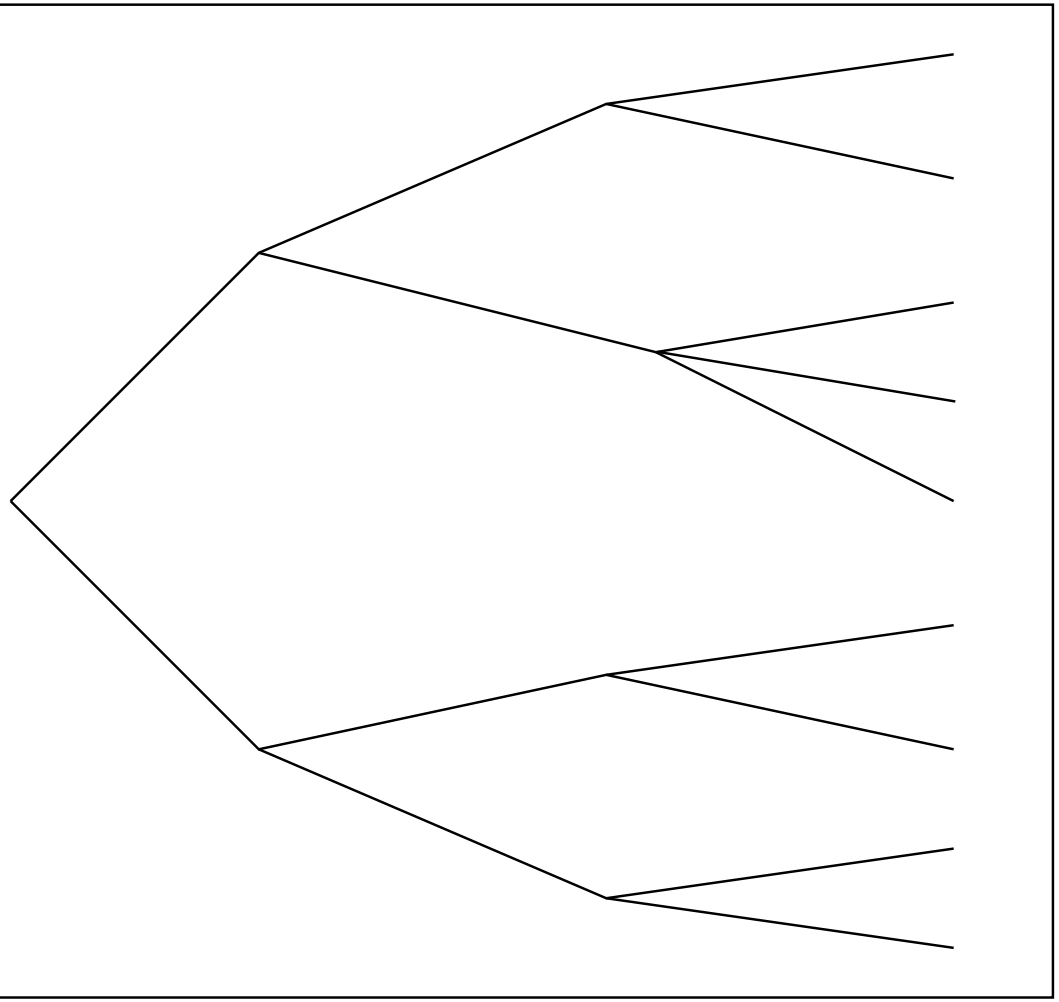


We may also define Finally and Globally in terms of Until:



Branching Time Logic (CTL*)

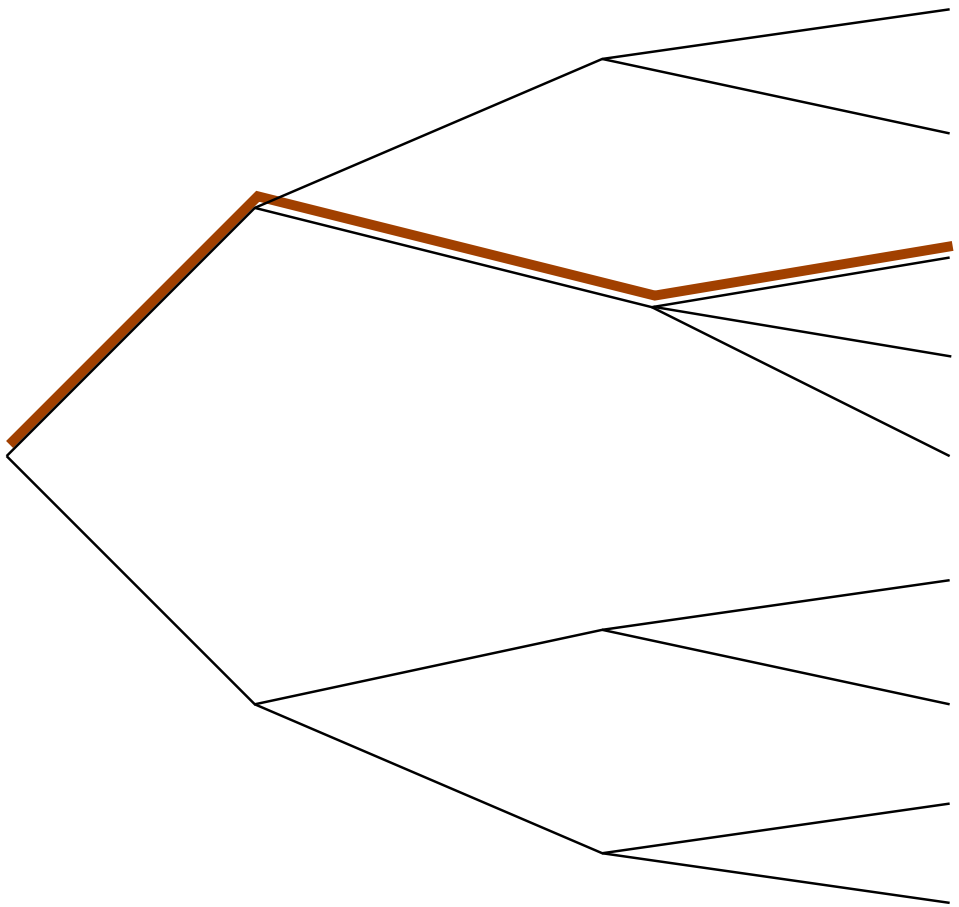
Structure M



Branching Time Logic (CTL*)

Structure M

Path σ

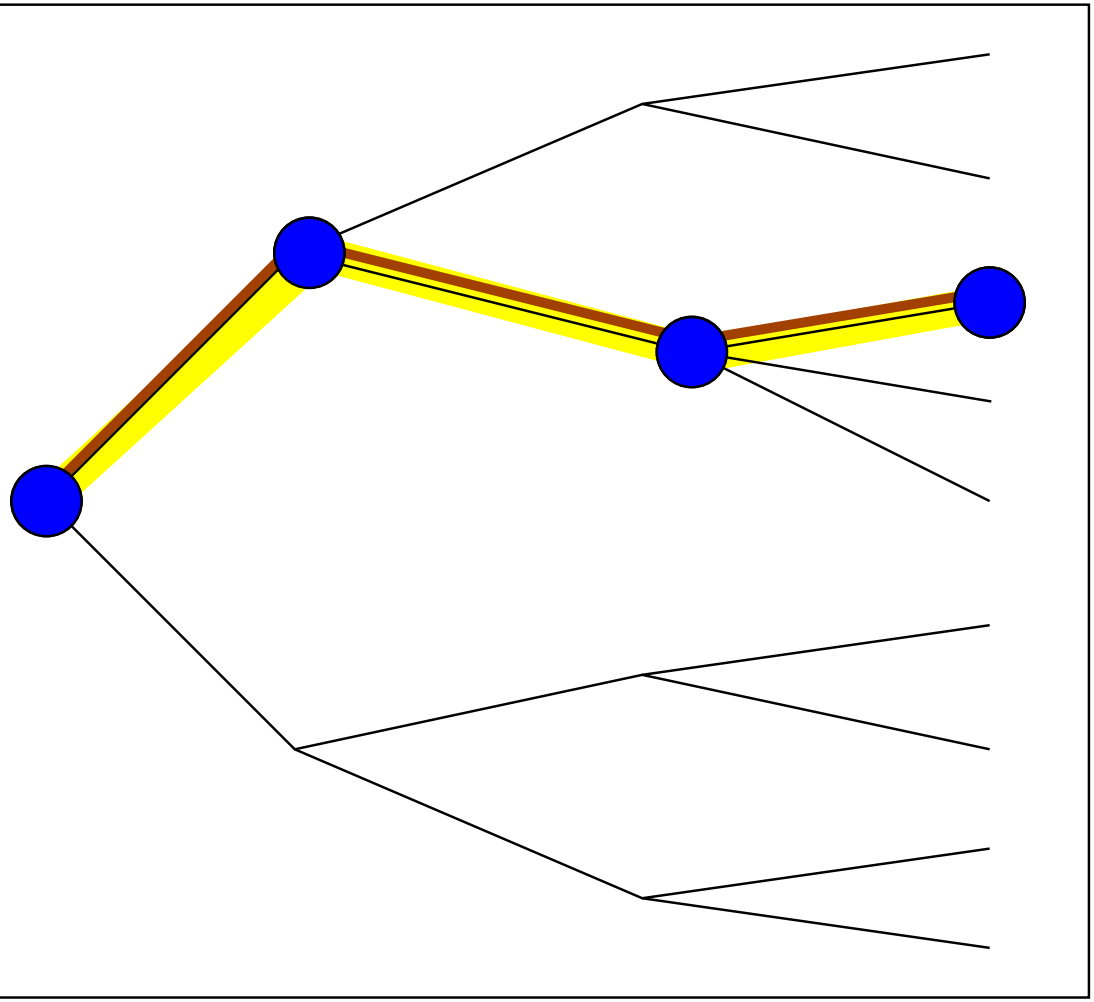


Branching Time Logic (CTL*)

Structure M

Path σ

Gb

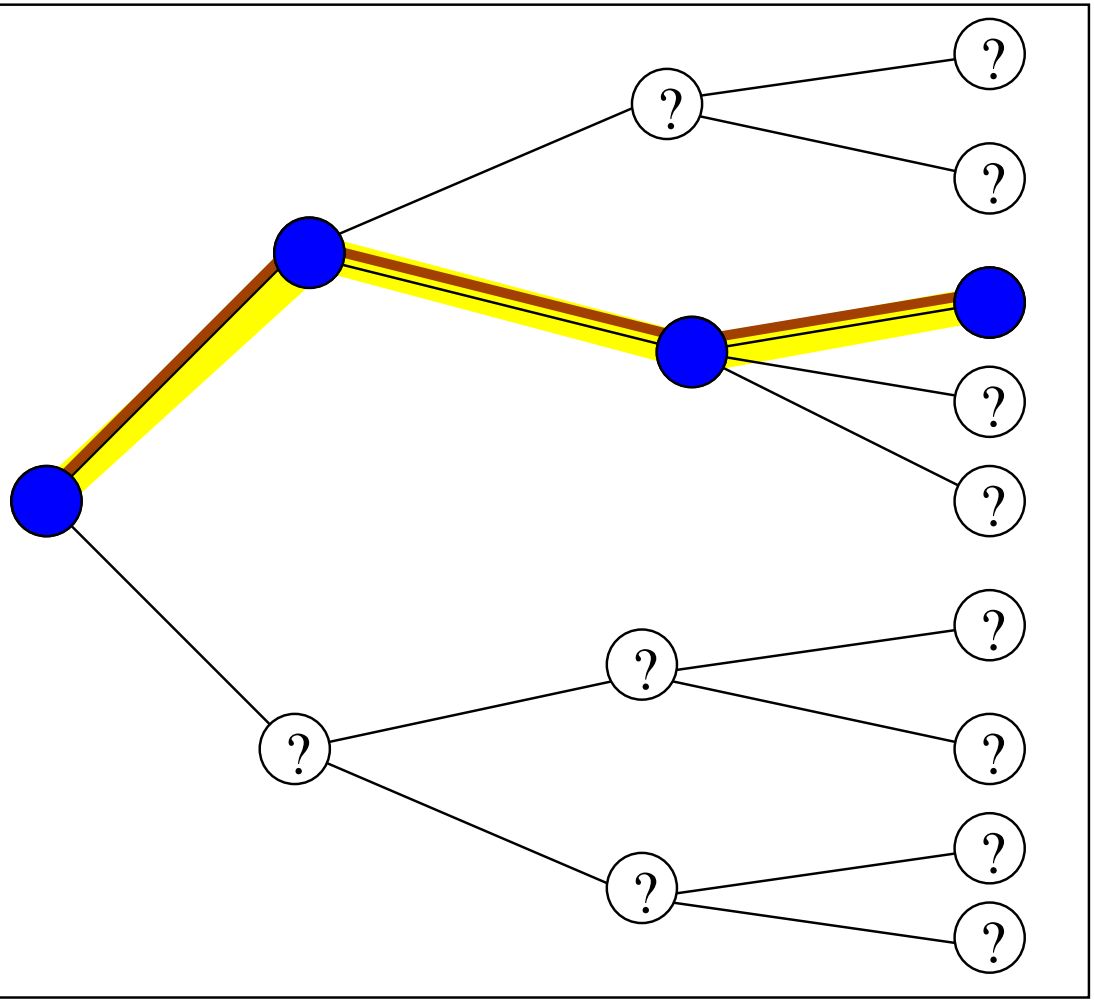


Branching Time Logic (CTL*)

Structure M

Path σ

Gb

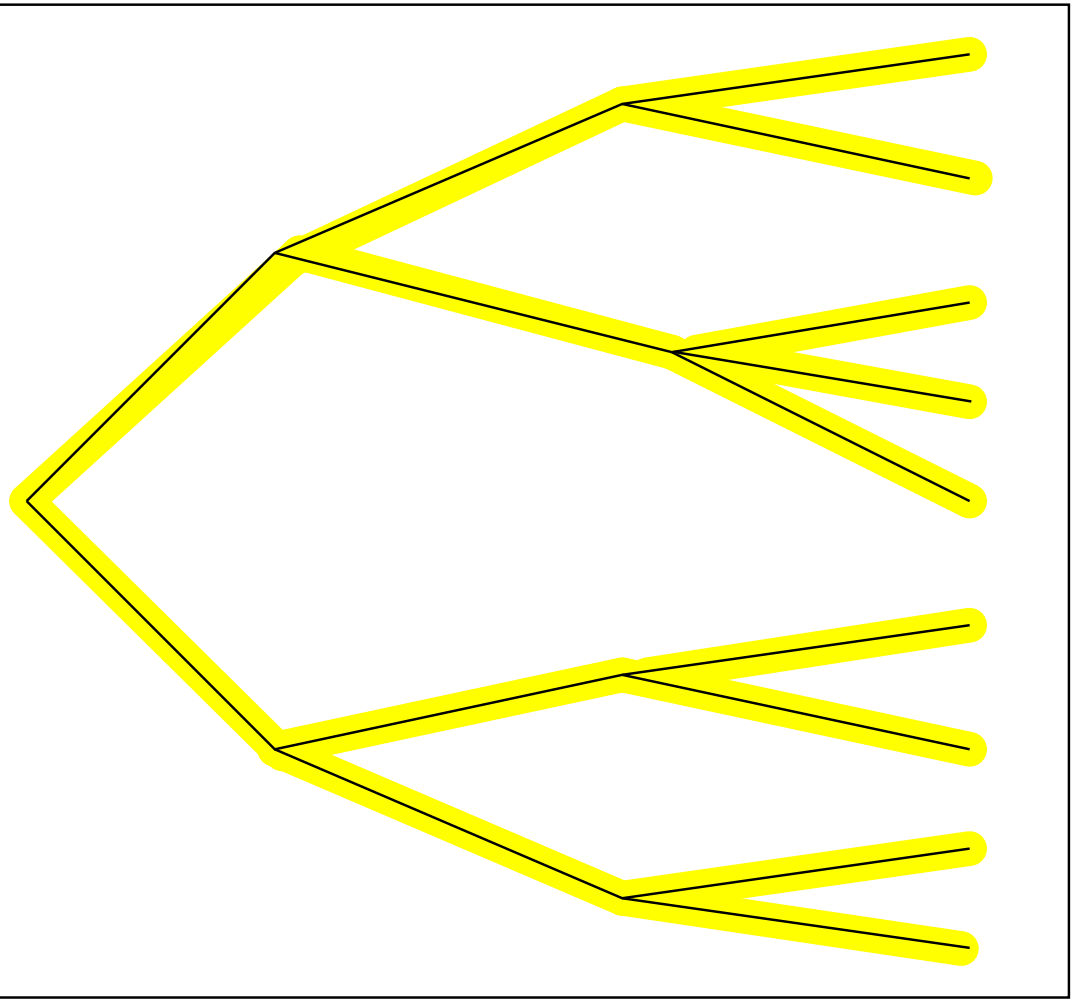


Branching Time Logic (CTL*)

Structure M

Path σ

(A)ll paths



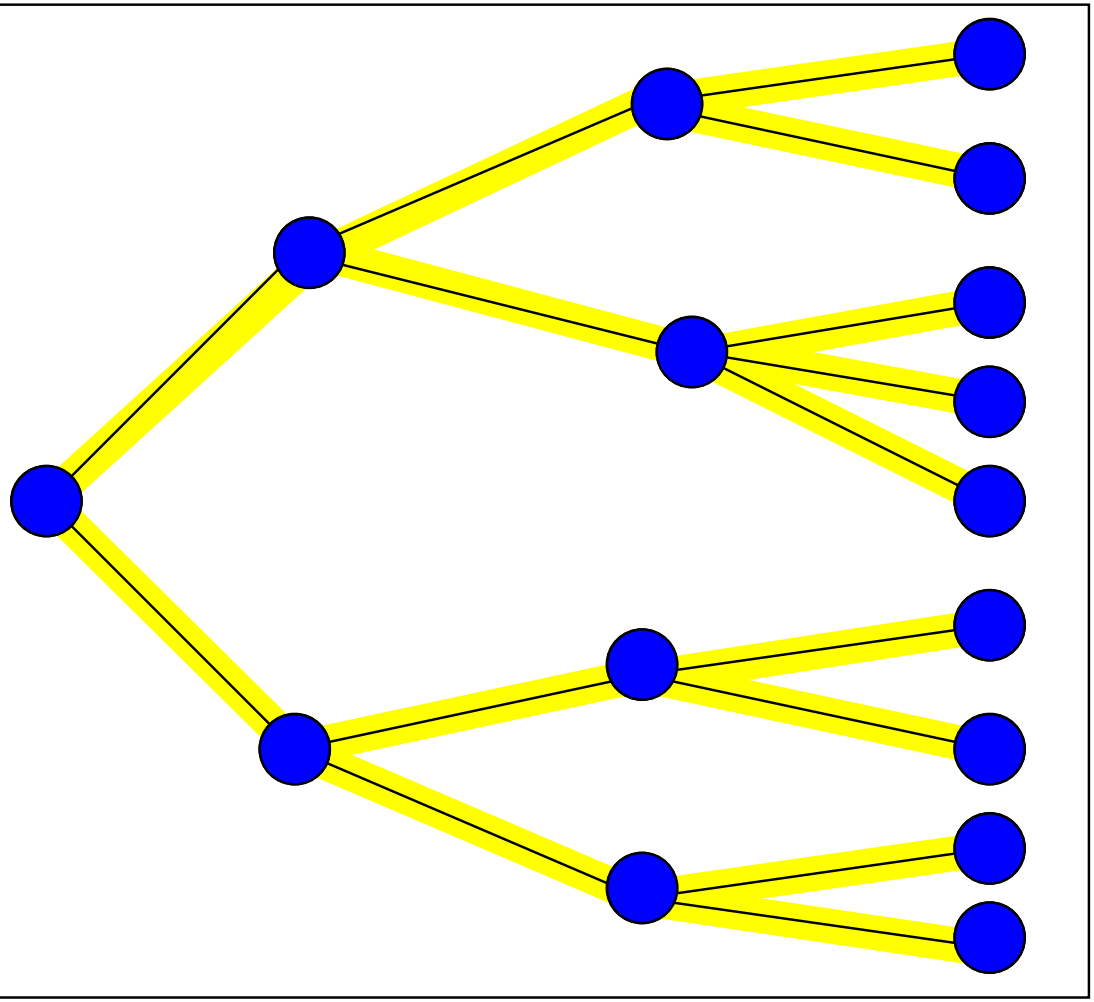
Branching Time Logic (CTL*)

Structure M

Path σ

(A)ll paths

AGb



Branching Time Logic (CTL*)

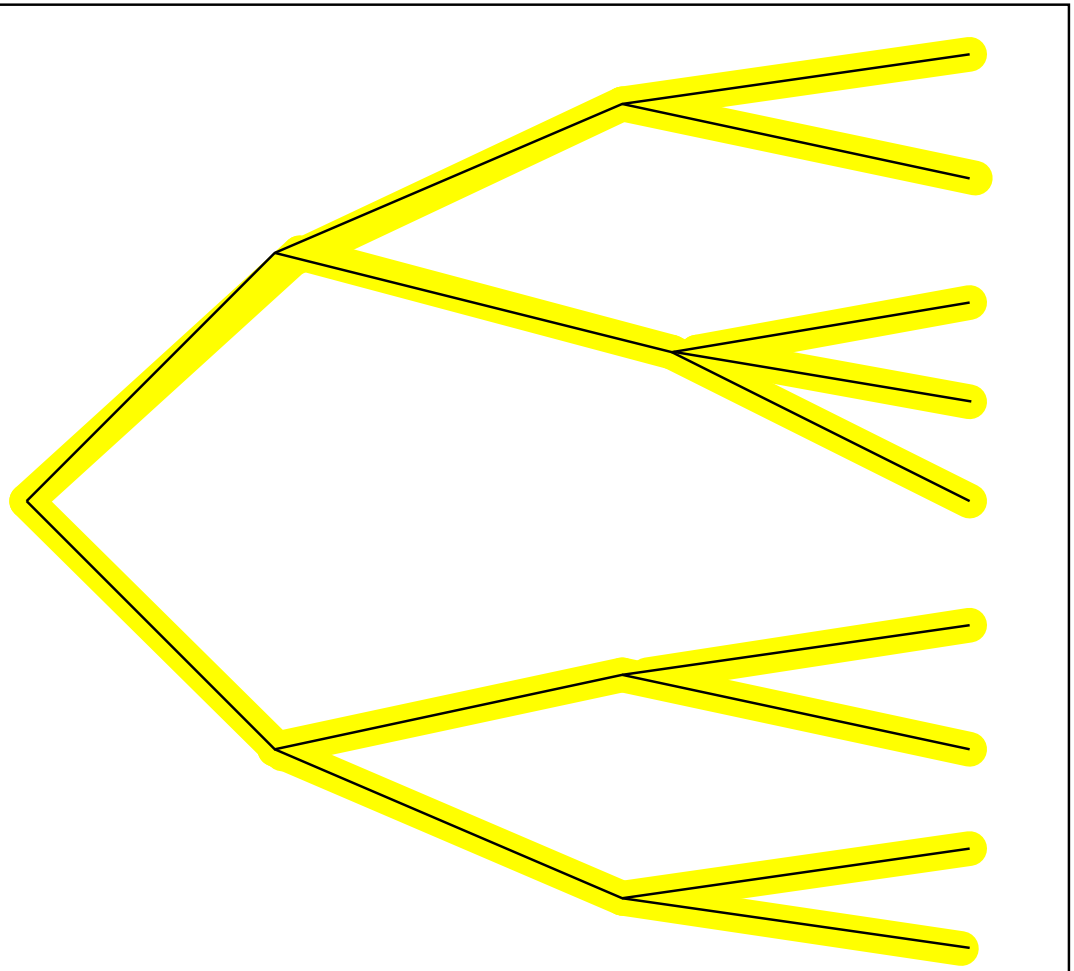
Structure M

Path σ

(A)ll paths

(E)xists a path

$E \equiv \neg A \neg$



Branching Time Logic (CTL*)

Structure M

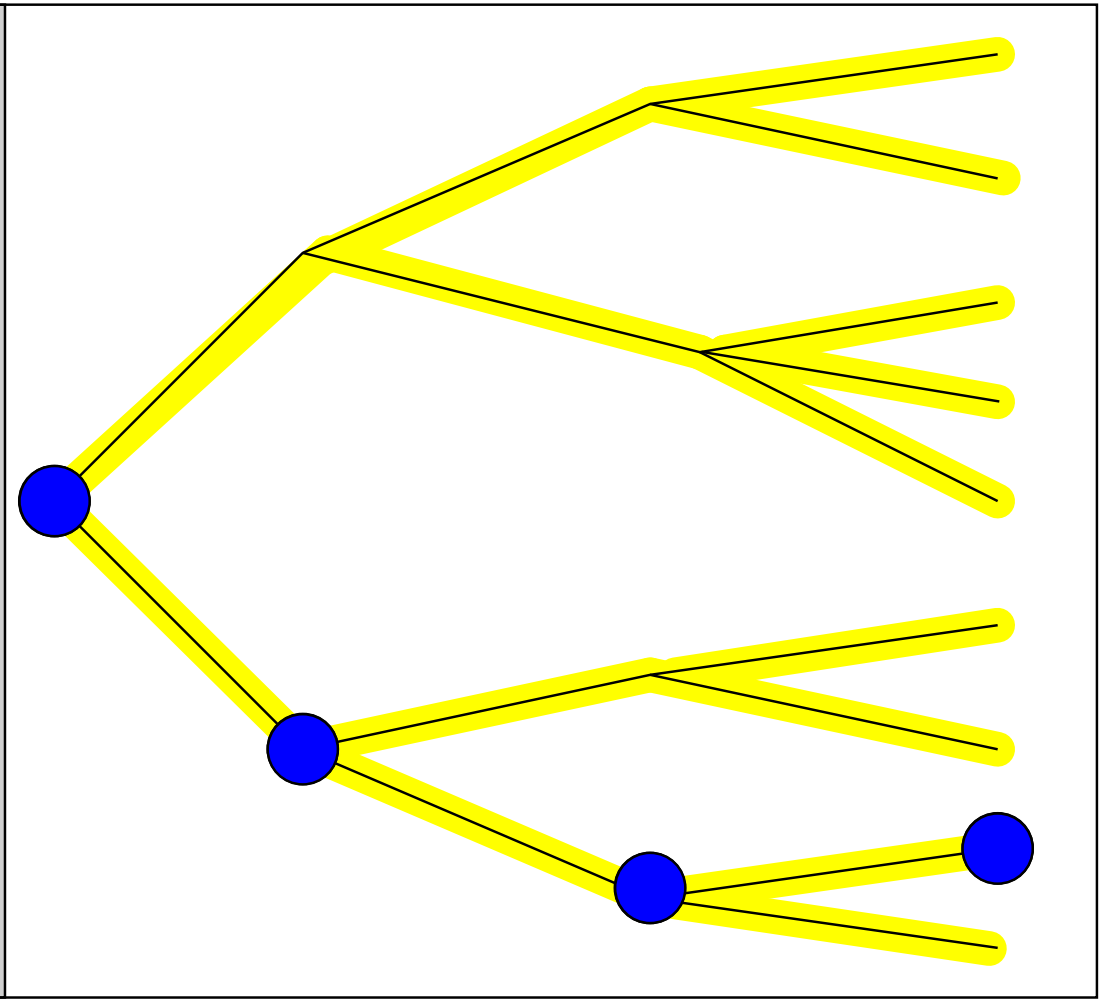
Path σ

(A)ll paths

(E)xists a path

$E \equiv \neg A \neg$

EGb



Branching Time Logic (CTL*)

Structure M

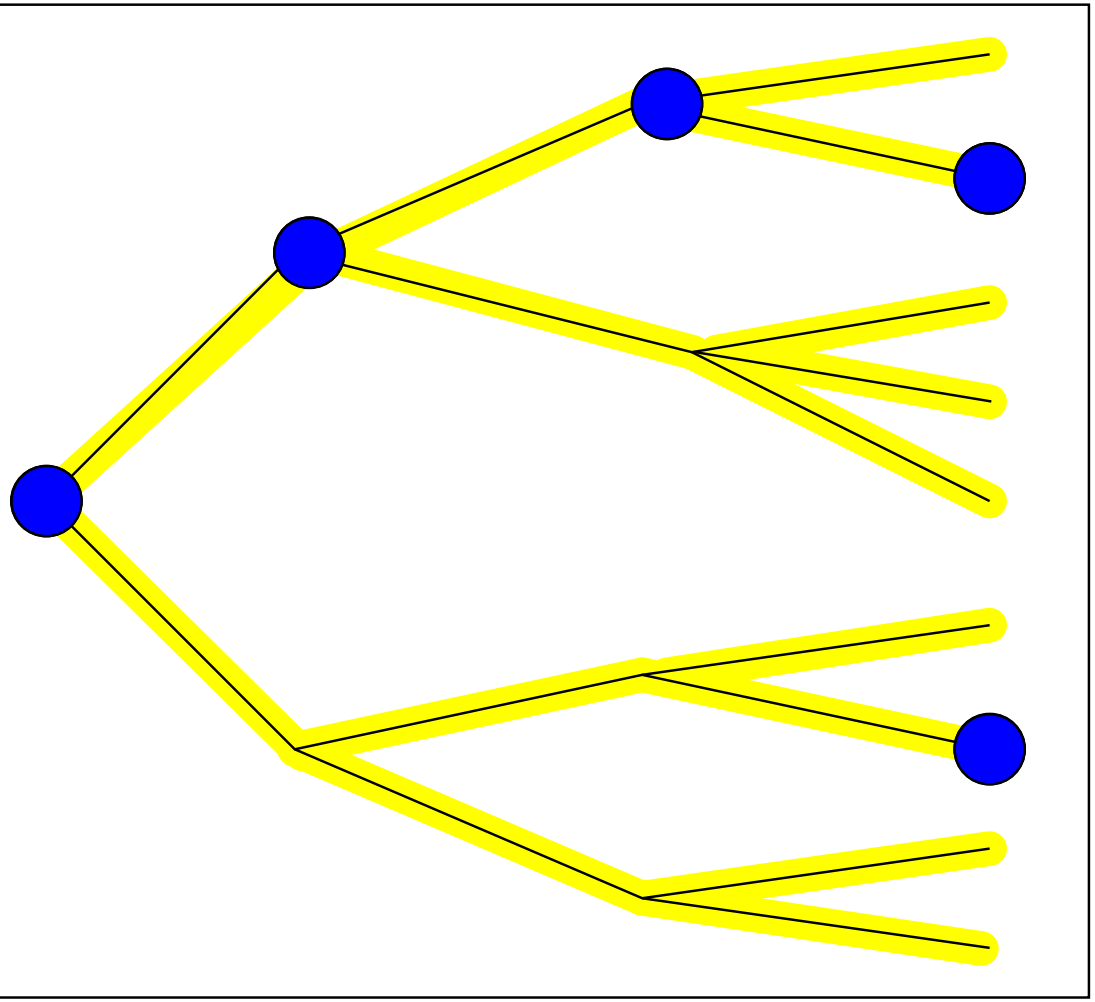
Path σ

(A)ll paths

(E)xists a path

$E \equiv \neg A \neg$

EGb



Branching Time Logic (CTL*)

Structure M

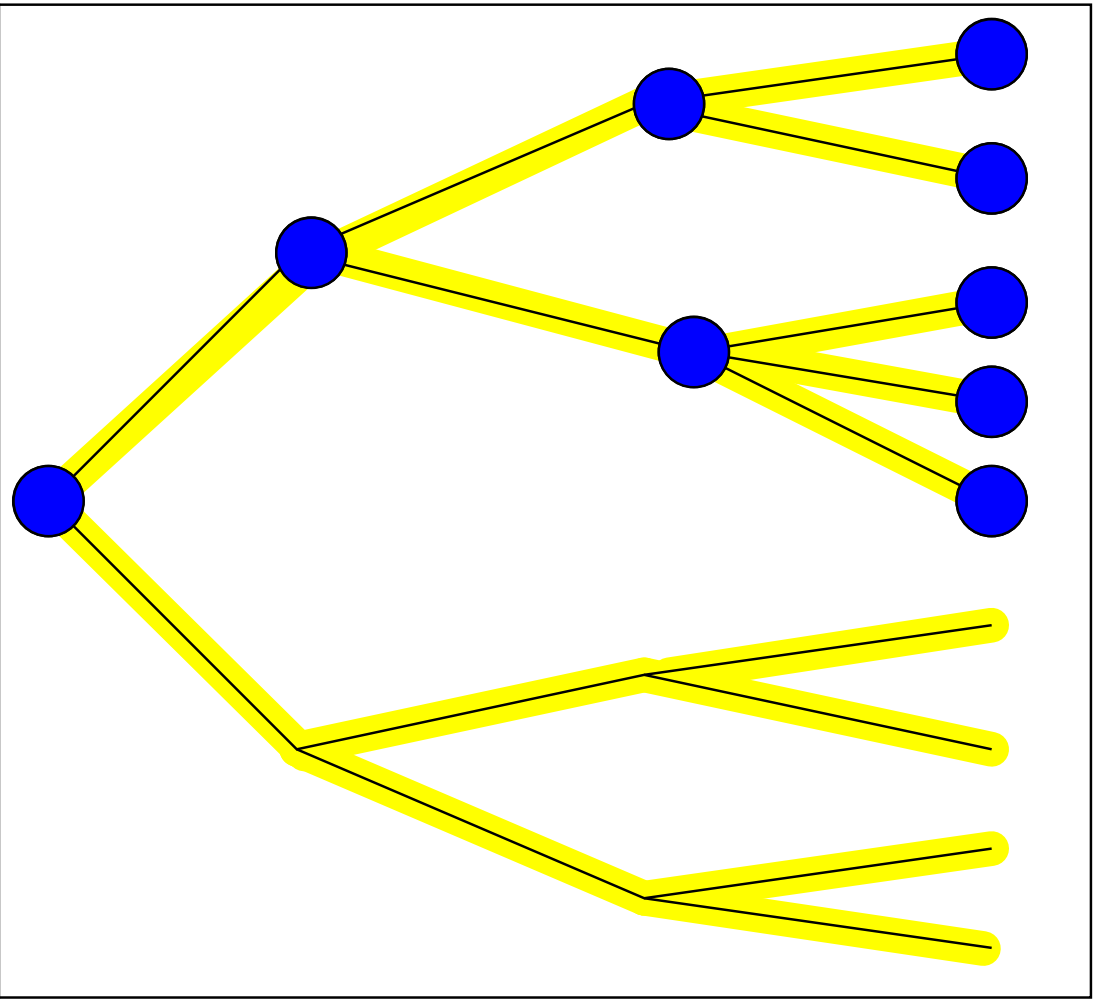
Path σ

(A)ll paths

(E)xists a path

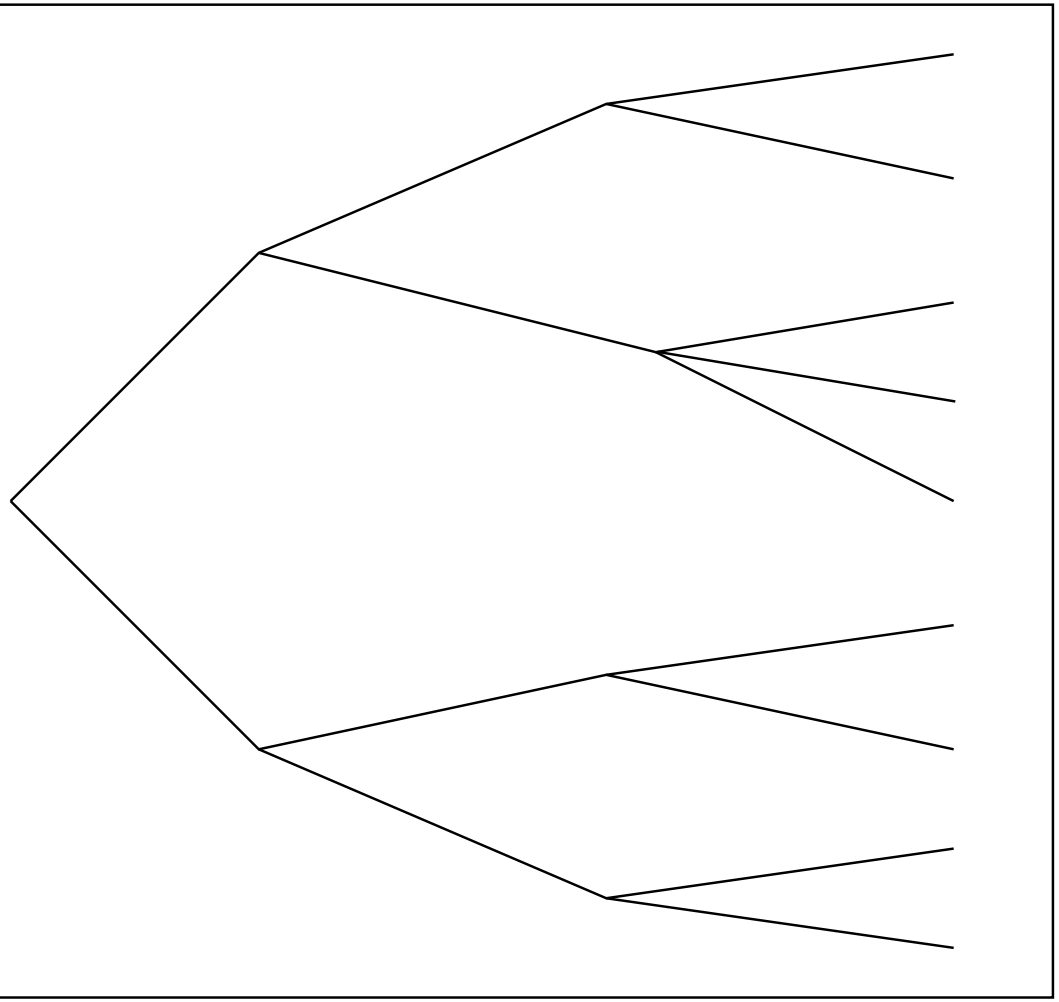
$E \equiv \neg A \neg$

EGb



Path Quantifier: Obligatory

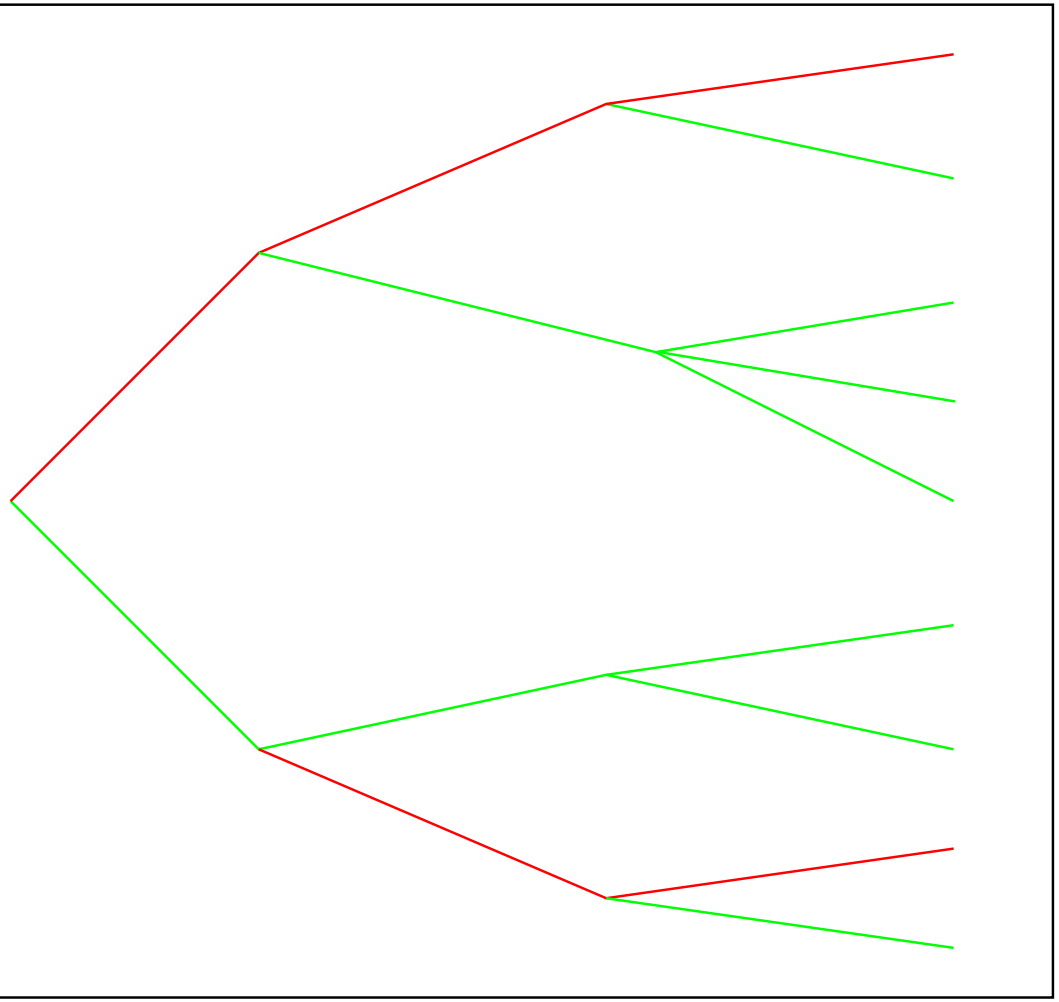
Structure M



Path Quantifier: Obligatory

Structure M

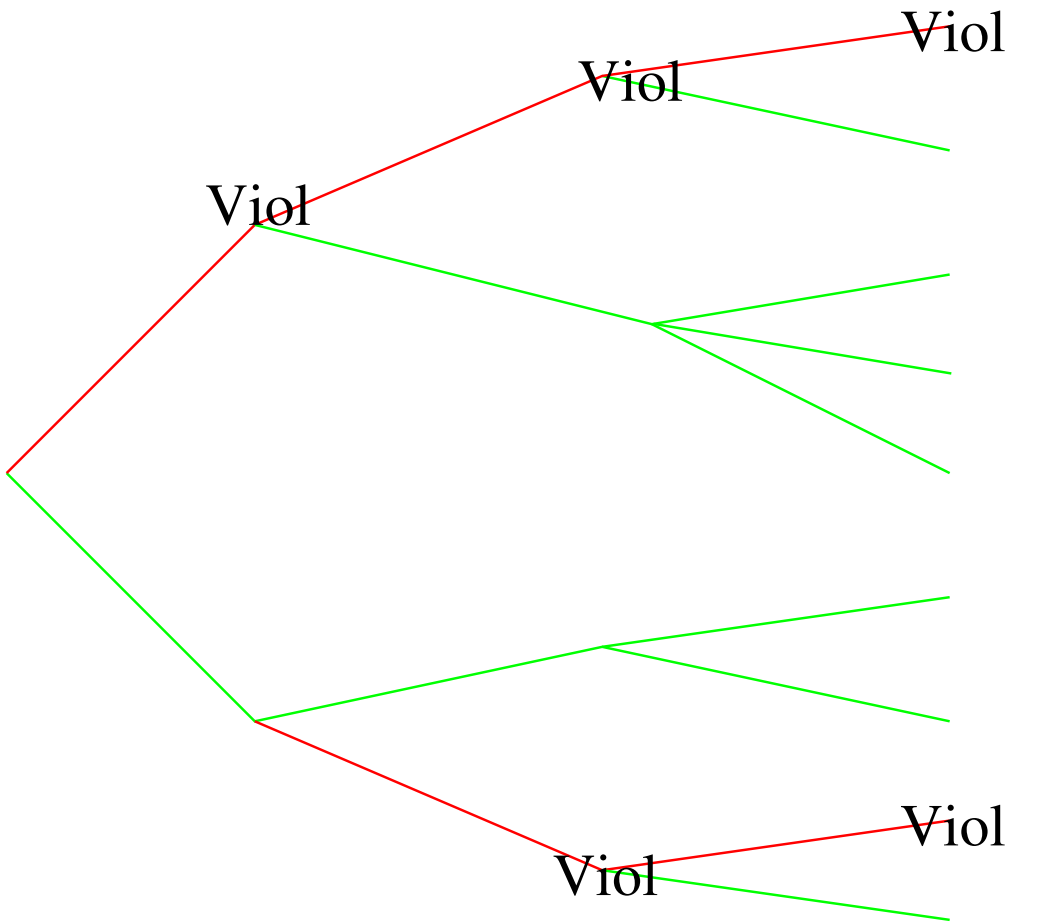
Success / Failure



Path Quantifier: Obligatory

Structure *M*

Success / Failure



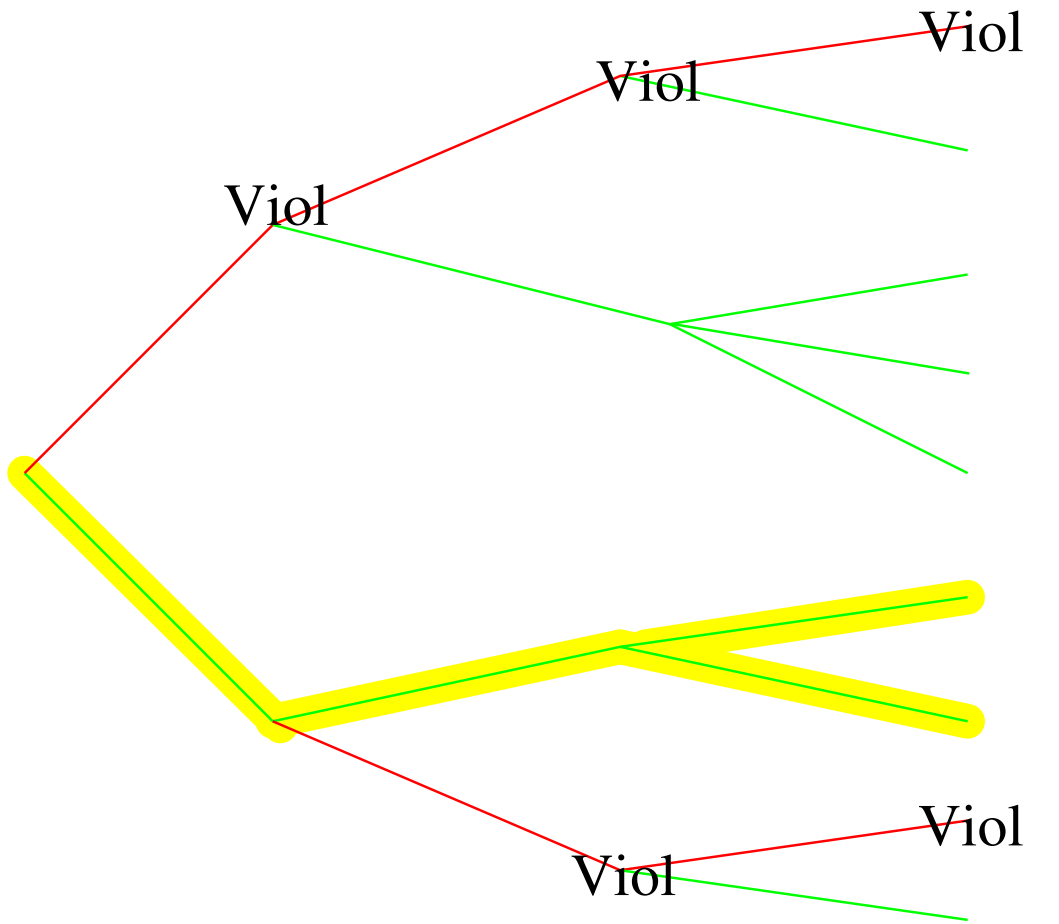
Path Quantifier: Obligatory

Structure M

Success/Failure

(*O*)bligatory

Green paths



Path Quantifier: Obligatory

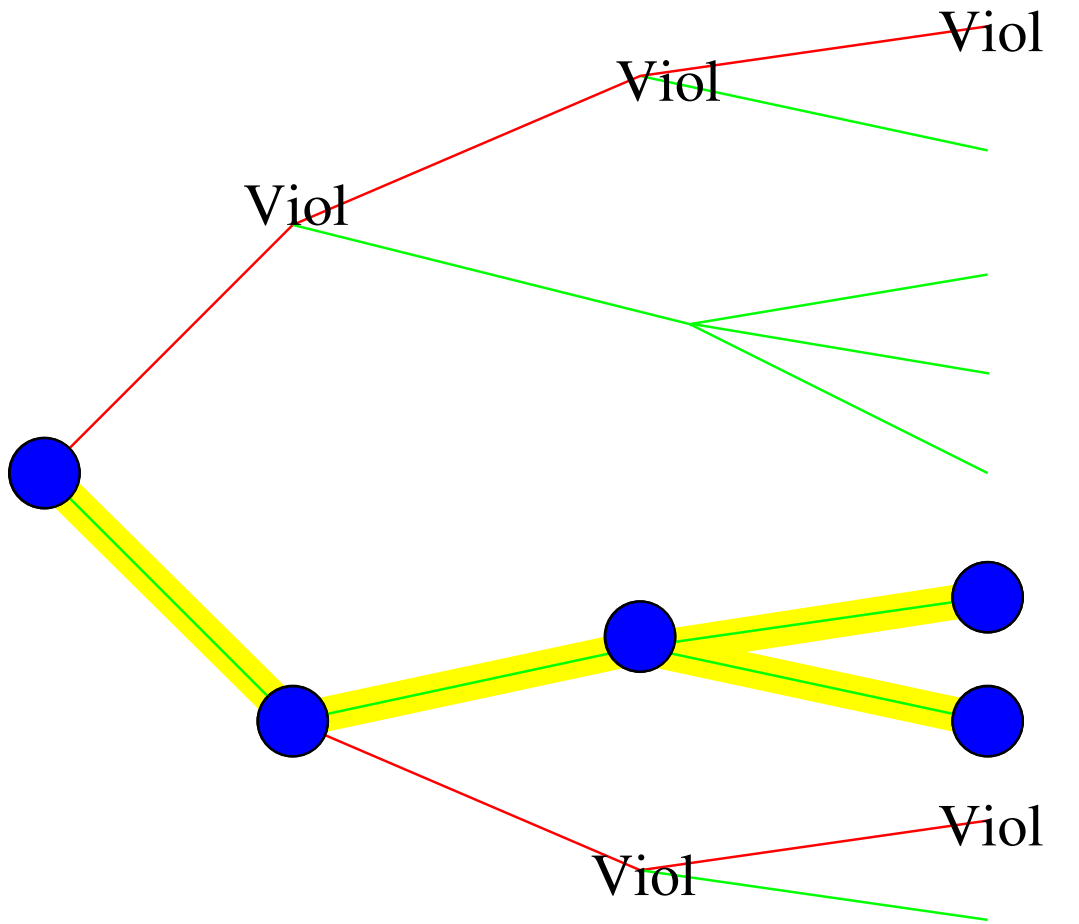
Structure *M*

Success/Failure

(*O*)bligatory

Green paths

OGb



Path Quantifier: Obligatory

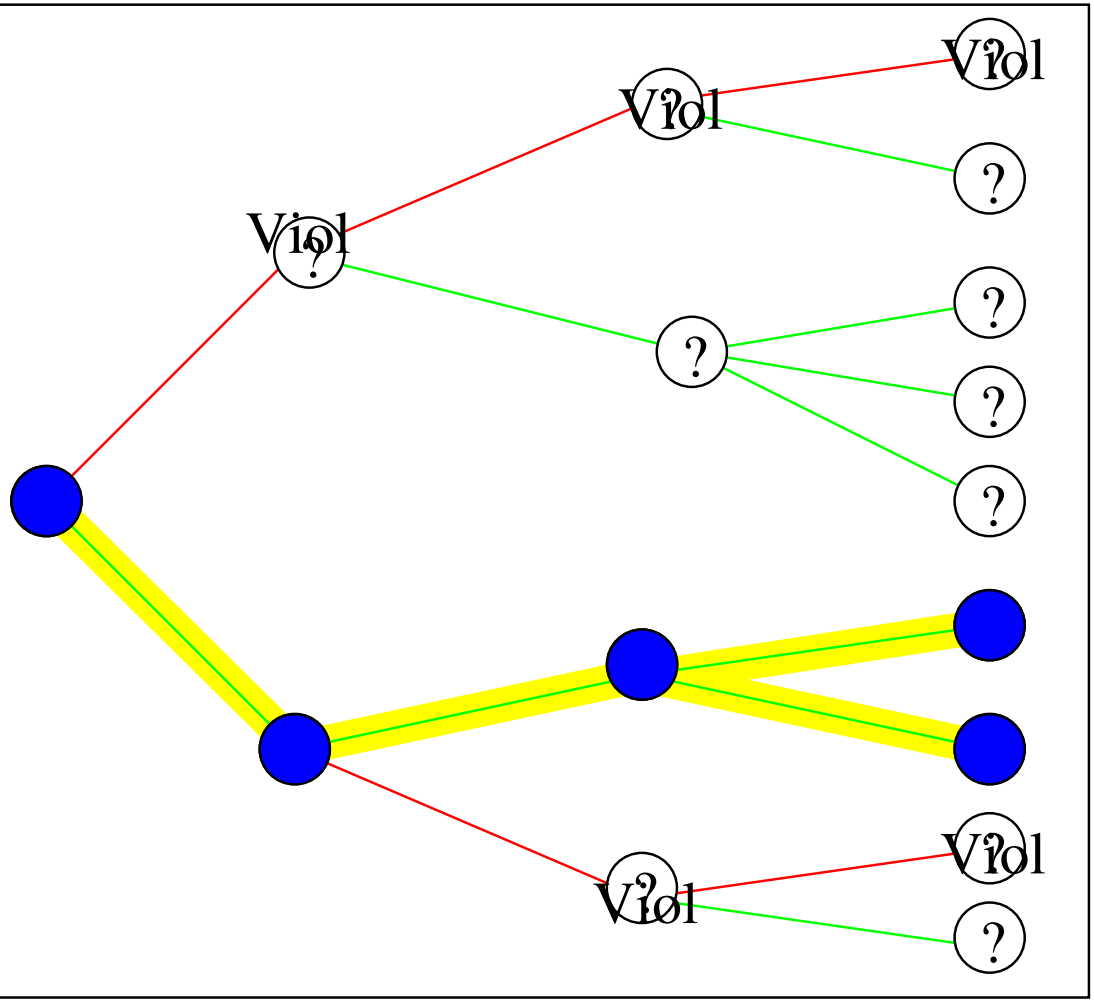
Structure M

Success/Failure

(O)bligatory

Green paths

OGb



Path Quantifier: Obligatory

Structure M

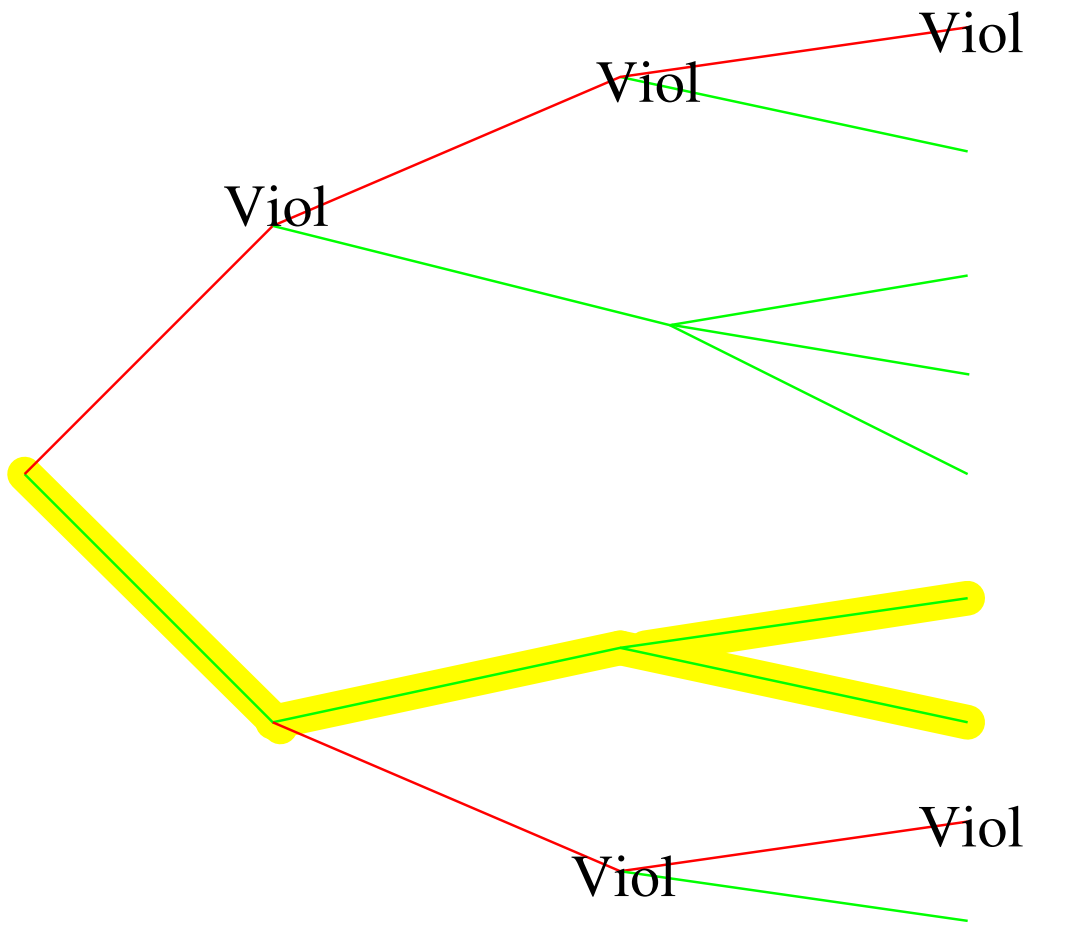
Success/Failure

(O)bligatory

Green paths

(P)ermissible

$P \equiv \neg O \neg$



Path Quantifier: Obligatory

Structure M

Success/Failure

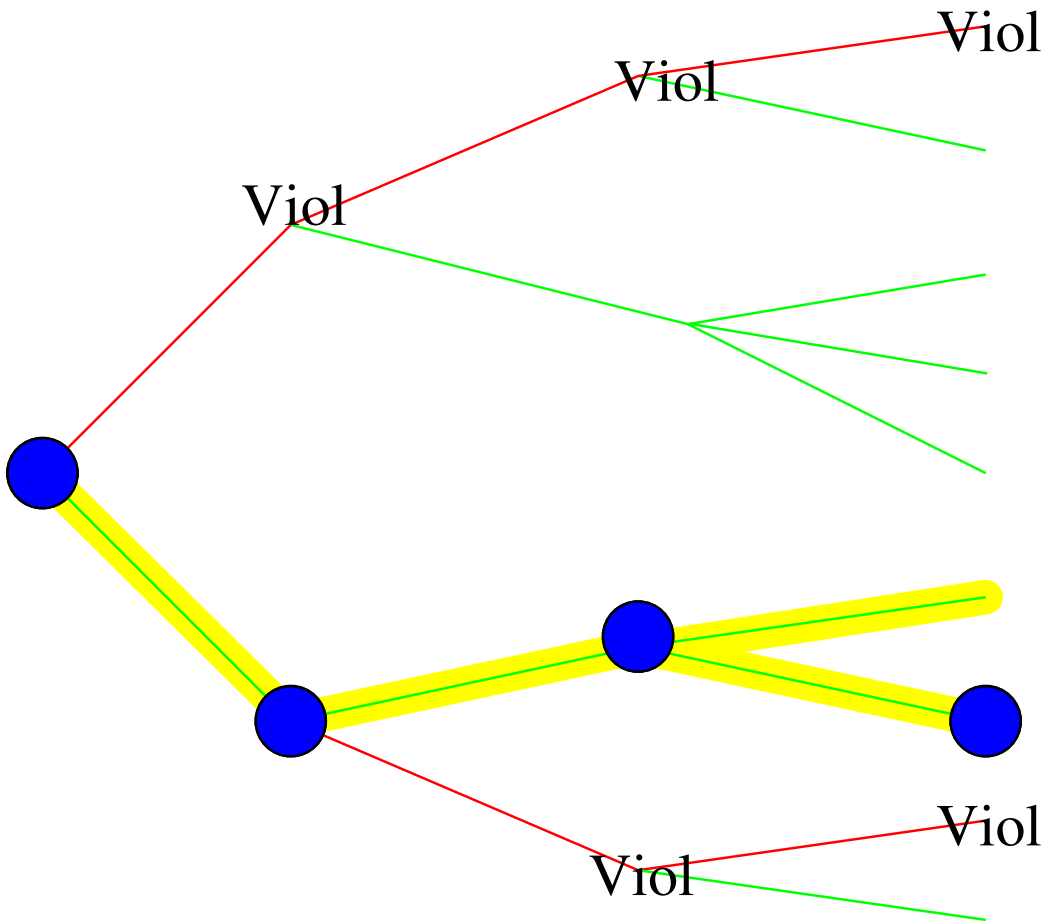
(O)bligatory

Green paths

(P)ermissible

$P \equiv \neg O \neg$

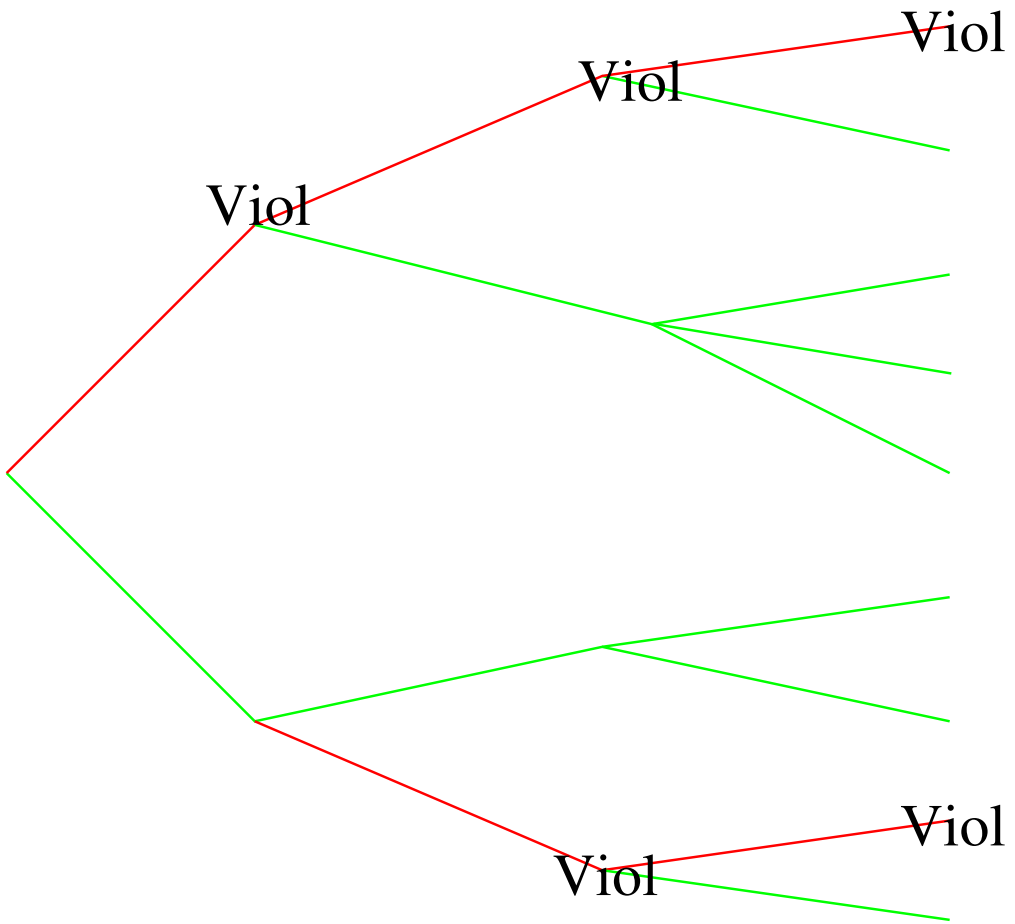
PGb



Path Quantifier: Robustly

Structure M

Robustly ($\blacktriangle$)



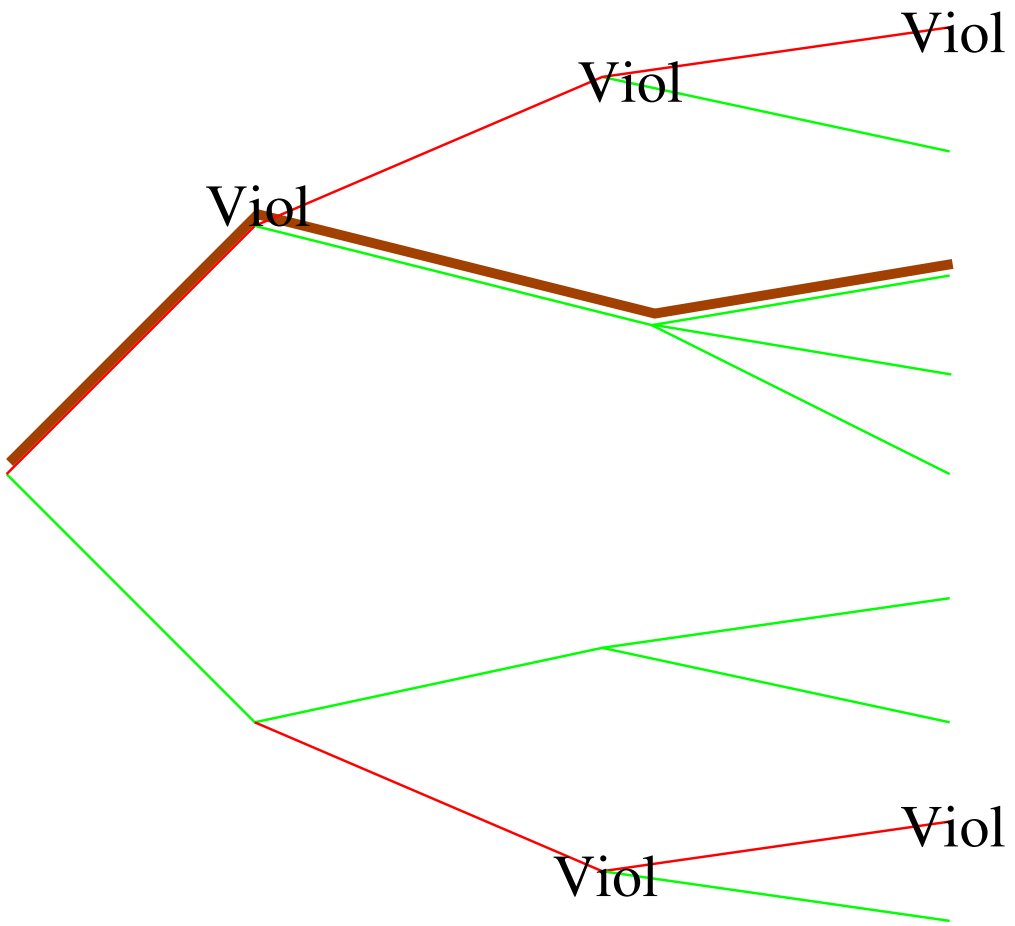
Path Quantifier: Robustly

Structure M

Robustly ($\blacktriangle$)

Path σ

$\blacktriangle \approx +1$ failure



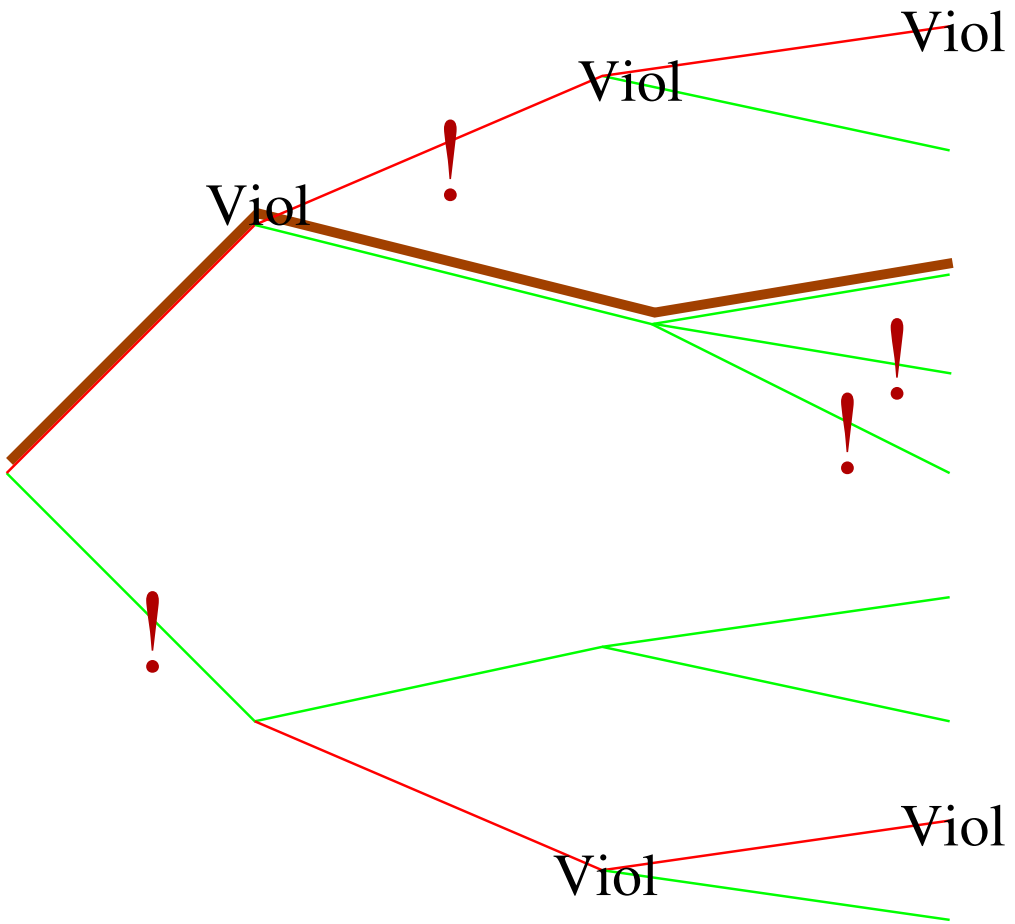
Path Quantifier: Robustly

Structure M

Robustly (▲)

Path σ

▲ $\approx$ +1 failure



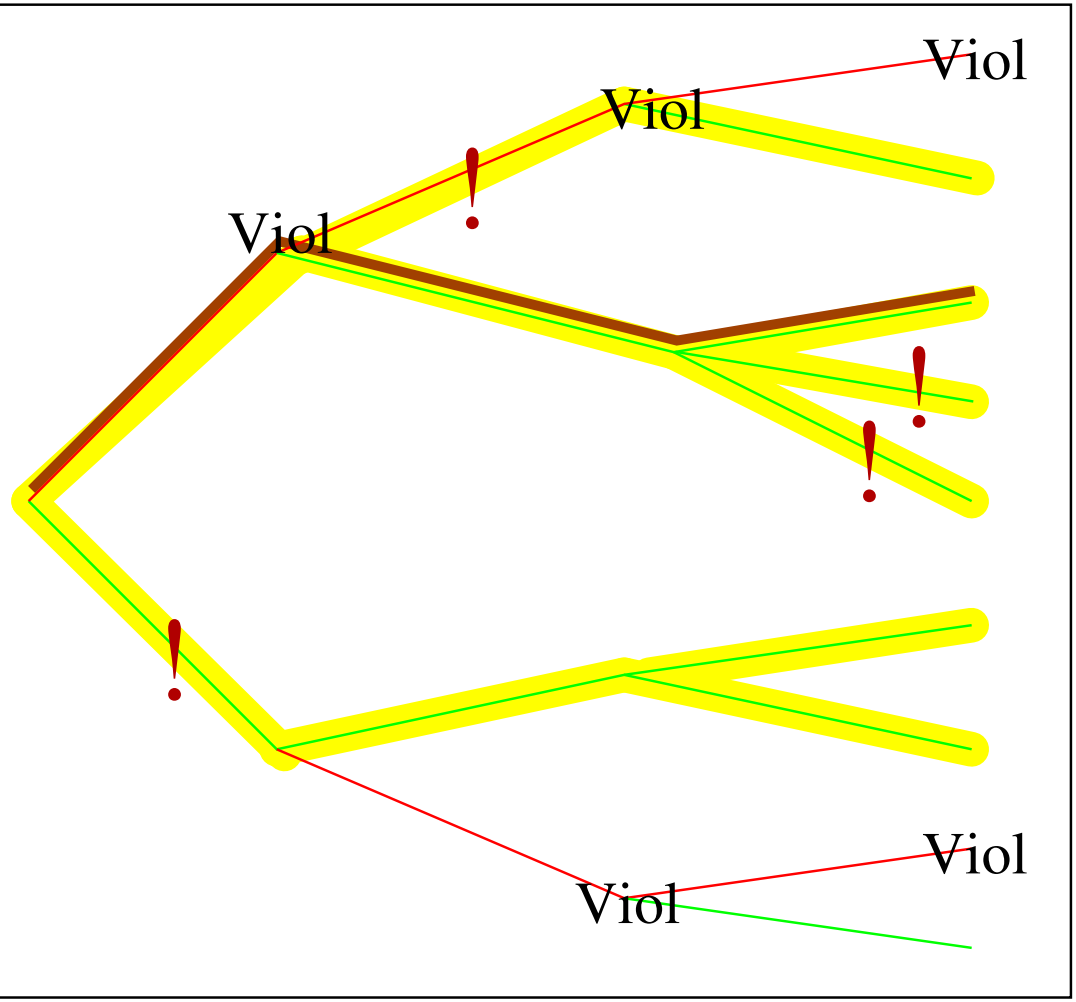
Path Quantifier: Robustly

Structure M

Robustly (▲)

Path σ

▲ $\approx$ +1 failure



Path Quantifier: Robustly

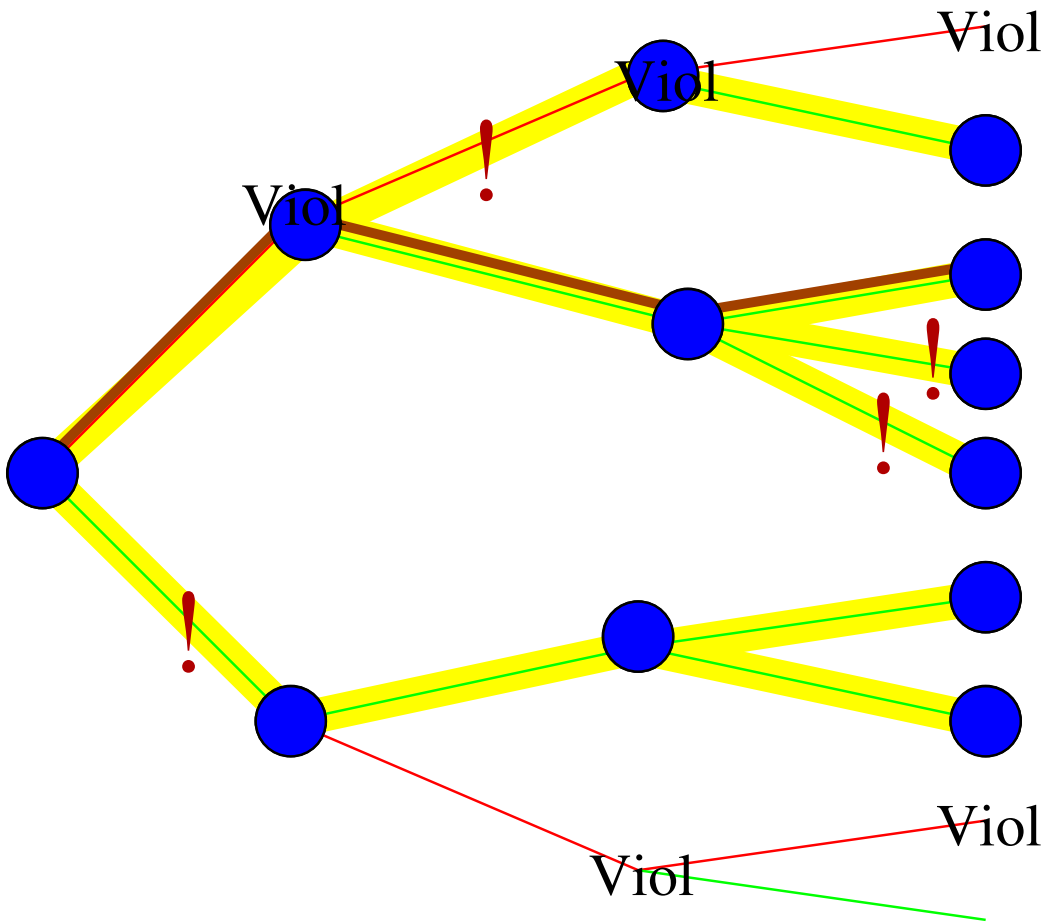
Structure M

Robustly ($\blacktriangle$)

Path σ

$\blacktriangle \approx$ +1 failure

$\blacktriangle Gb$



Path Quantifier: Robustly

Structure M

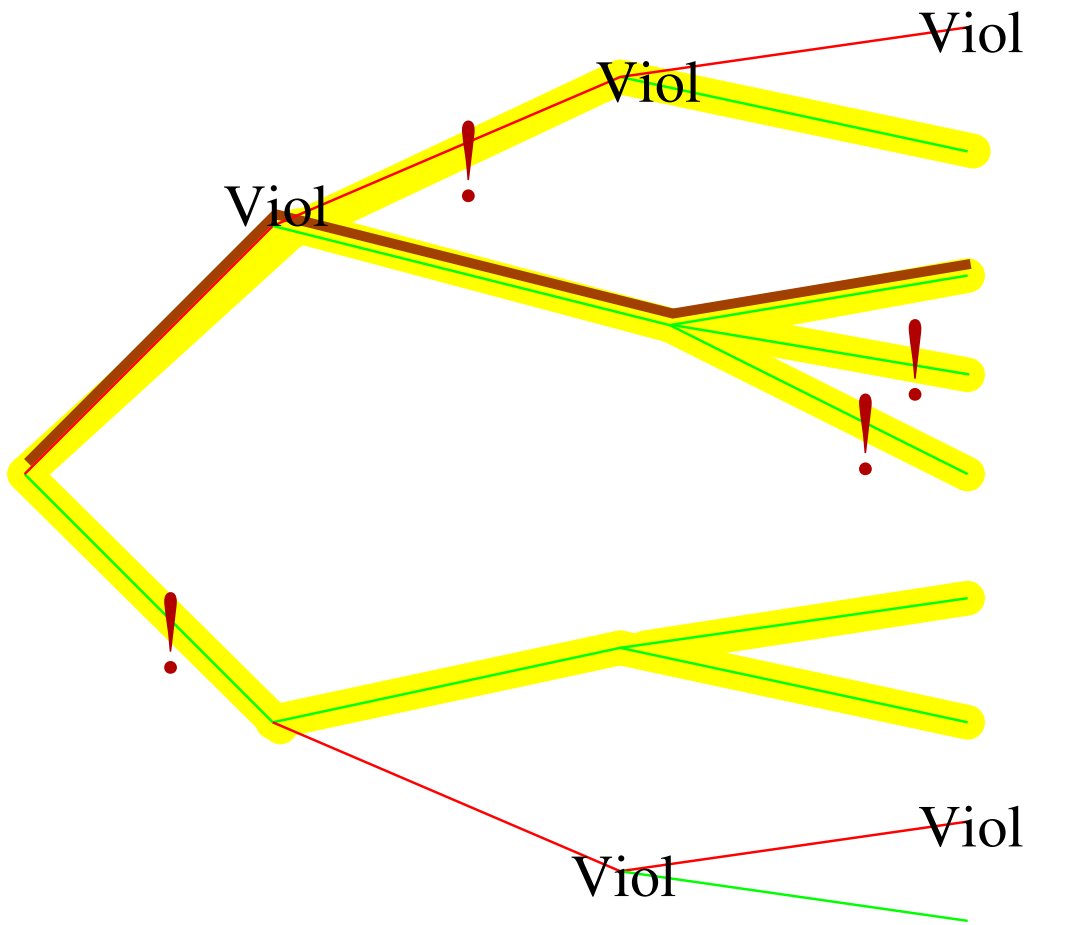
Robustly ($\blacktriangle$)

Path σ

$\blacktriangle \approx +1$ failure

Prone (Δ)

$\Delta \equiv \neg\blacktriangle\neg$



Path Quantifier: Robustly

Structure M

Robustly ($\blacktriangle$)

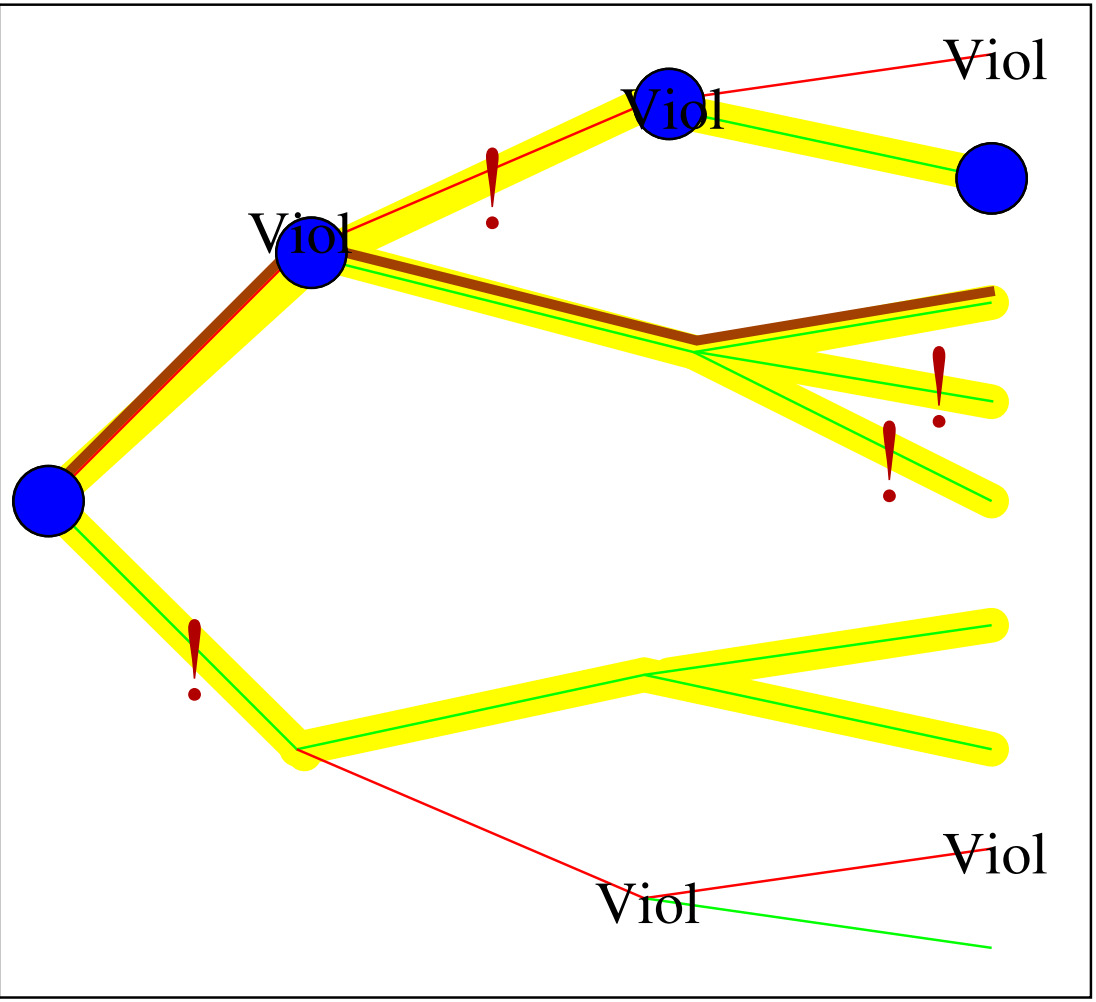
Path σ

$\blacktriangle \approx$ +1 failure

Prone (Δ)

$\Delta \equiv \neg\blacktriangle\neg$

$\Delta G b$



Path Quantifier: Robustly

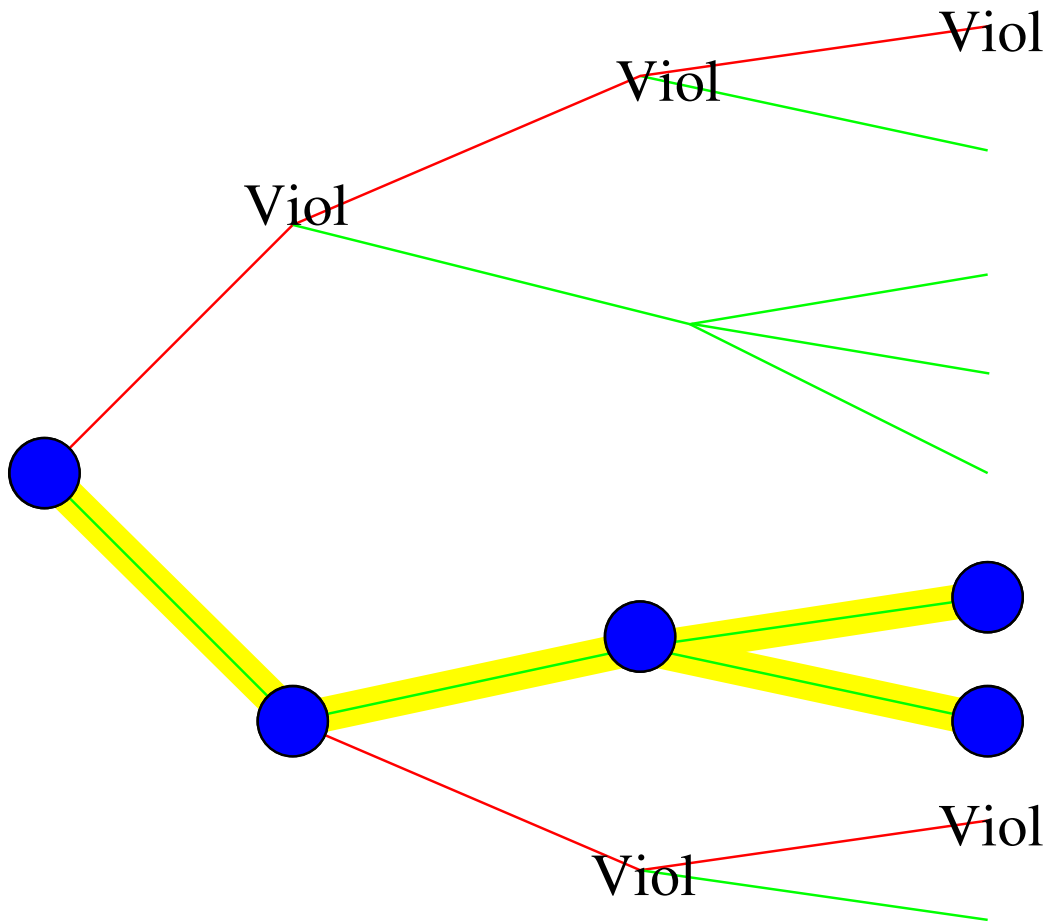
Structure M

Robustly ($\blacktriangle$)

OGb

0 failures

$\blacktriangle \approx +1$ failure



Path Quantifier: Robustly

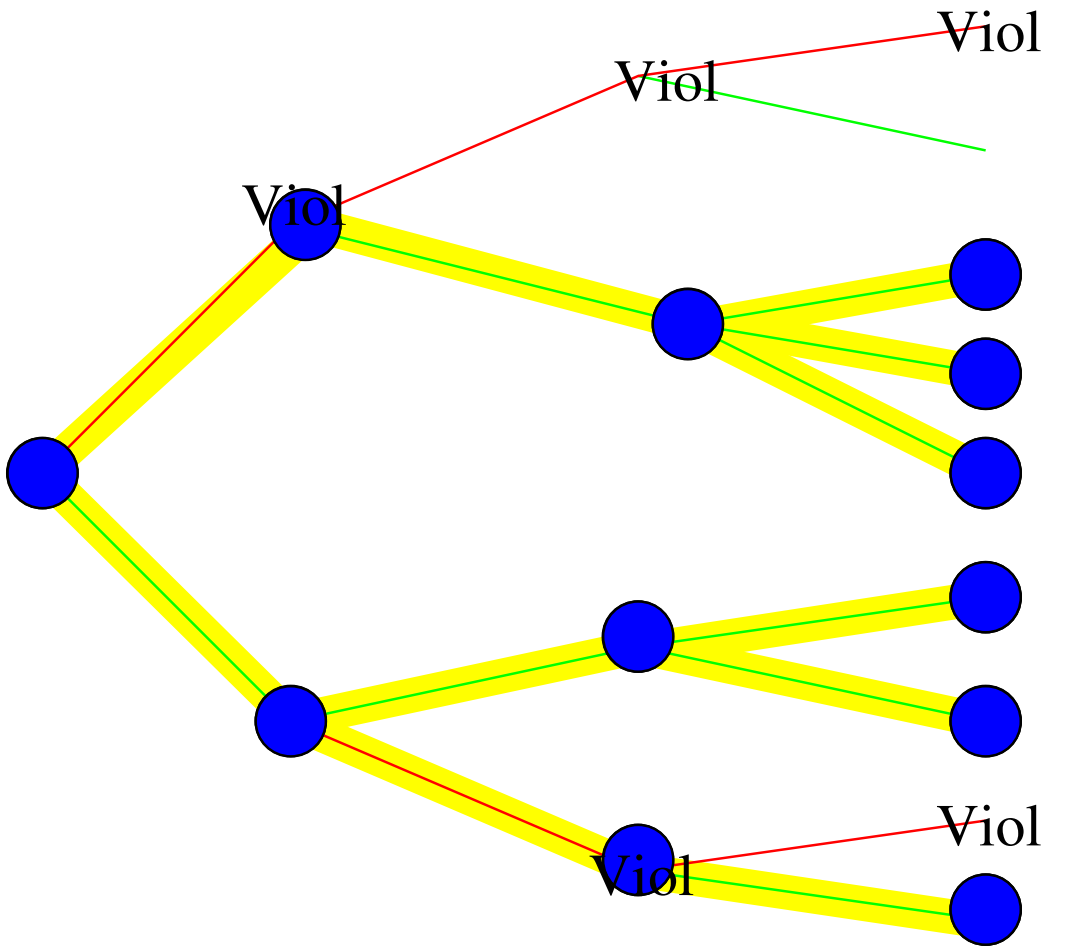
Structure M

Robustly ($\blacktriangle$)

$O \blacktriangle Gb$

≤ 1 failures

$\blacktriangle \approx +1$ failure



Path Quantifier: Robustly

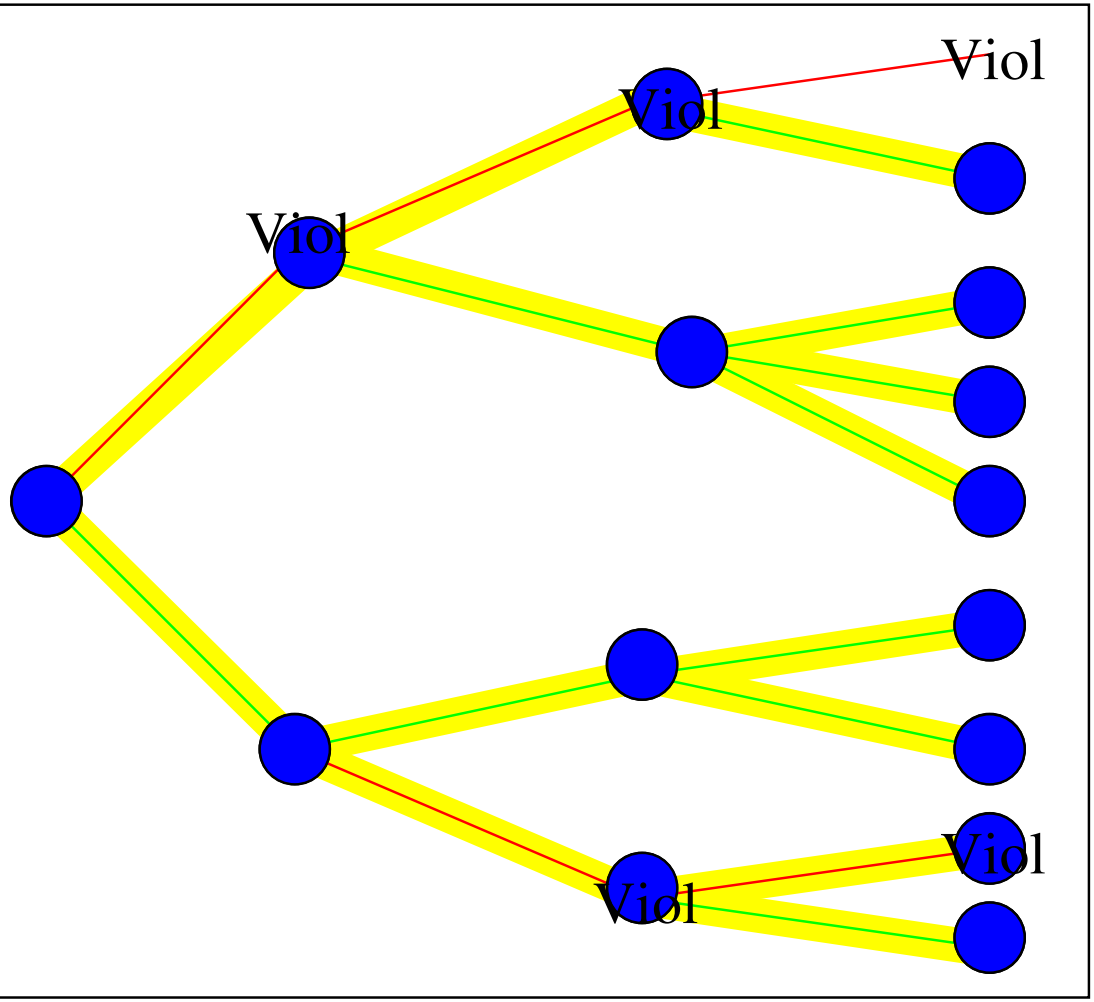
Structure M

Robustly ($\blacktriangle$)

$O\blacktriangle\blacktriangle Gb$

$\leq$ 2 failures

$\blacktriangle \approx$ +1 failure

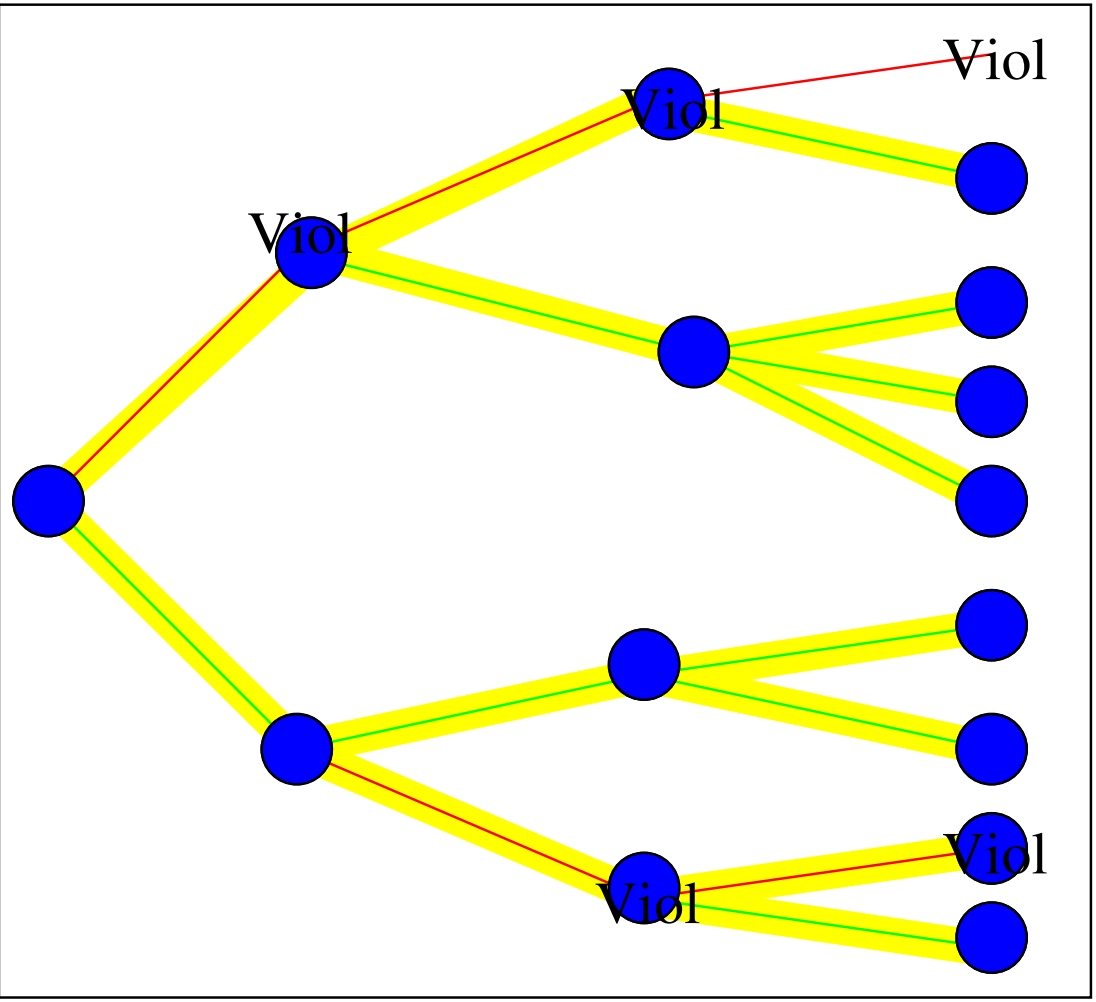


Path Quantifier: Robustly

Structure M

Robustly ($\blacktriangle$)

$\blacktriangle \approx +1$ failure



Bundles

In (Ro)CTL* any path through structure is allowed.
In (Ro)BCTL* we may be limited to a bundle of paths.

- No Limit Closure axiom.

Consider a path through a maze:

- CTL*: If you can always turn left, there is a path consisting only of left turns.
- BCTL*: It may be case that you always have option of turning left, but all paths must have at least one right turn, eventually.

Semantics

We limit paths to some bundle of paths B

$\delta(\sigma)$: Deviations from σ, for $\tau \in B$

$\tau \in \delta(\sigma)$ iff $\exists i$ st $\tau_{\leq i} = \sigma$ and $\tau_{\geq i+1}$ is failure-free

$\mathcal{SF}(s)$ paths in B starting at s

$\mathcal{S}(s)$ failure-free paths in B starting at s

$M, \sigma \vDash \phi$ means ϕ is true on the structure M along path σ

$$M, \sigma \vDash N\phi \text{ iff } M, \sigma_{\geq 1} \vDash \phi$$

$$M, \sigma \vDash \phi U \psi \text{ iff } \exists_{i \in \mathbb{N}} \text{ s.t. } M, \sigma_{\geq i} \vDash \psi \text{ and } \forall_{j \in \mathbb{N}} j < i \implies M, \sigma_{\geq j} \vDash \psi$$

$$M, \sigma \vDash A\phi \text{ iff } \forall_{\pi \in \mathcal{SF}(\sigma_0)} M, \pi \vDash \phi$$

$$M, \sigma \vDash O\phi \text{ iff } \forall_{\pi \in \mathcal{S}(\sigma_0)} M, \pi \vDash \phi$$

$$M, \sigma \vDash \blacktriangle\phi \text{ iff } \forall_{\pi \in \delta(\sigma)} M, \pi \vDash \phi \text{ and } M, \sigma \vDash \phi$$

Coordinated Attack Problem

General A wants to organise an attack with B

$AG(s_A \rightarrow ONr_B)$: If A sends a message, B should receive it at the next step.

$AG(\neg s_A \rightarrow \neg Nr_B)$: If A does not send a message now, B will not receive a message at the next step.

$AG(f_A \rightarrow AGf_A)$: Once A commits, A cannot withdraw.

$AG(f_A \rightarrow \neg s_A)$: If A has committed to an attack, it is too late to send messages.

$A(\neg f_A W r_A)A$ cannot commit to an attack until A has received plans from B

Coordinated Attack Protocol

For unbounded # of errors, no protocol exists, but…
Following works if one or less messages are lost.

$A(s_A W r_A)$: General A will send plans until a response is received.

$AG(r_A \rightarrow f_A)$: Once general A receives a response, A will commit to an attack.

$A(\neg r_B W (r_B \wedge (s_B \wedge N s_B \wedge N N f_B)))$: Once general B receives plans, B will send two messages to A and then commit to an attack.

Decide: is the conjunction $\hat{\phi}$ of the above formulæ consistent under Ro(B)CTL*?
Decide: does $\hat{\phi}$ imply correct behaviour even if a single failure occurs: $\hat{\phi} \rightarrow O\blacktriangle F(f_A \wedge f_B)$

Feeding A Cat.

We wish to ensure that a cat always has food (f) when it is hungry (h), even if we once forget to fill the cat bowl.

1. $AG(h \rightarrow \neg Nh)$: Not hungry twice in a row.
2. $AG((h \vee \neg f) \rightarrow \Delta N \neg f)$: May forget to fill empty food bowl.
3. $AG((\neg h \wedge f) \rightarrow Nf)$: If the cat is not hungry the bowl will not be emptied.
4. $O\blacktriangle G(h \rightarrow f)$: Even with single error, must be food whenever cat is hungry
5. f: The cat bowl starts full.

Decide: is policy is consistent?

Decide: $AGONf \rightarrow O\blacktriangle G(h \rightarrow f)$: Always attempting to fill cat bowl satisfies policy.

Overview of Tableau of BCTL*

- Paths $\longrightarrow$ Hues
  - represents formulae that could hold together on the same path.
  - is a set of formulae
- States $\longrightarrow$ Colours
  - Many paths can start at same state
  - Colours are sets of hues

Defined in terms of closure sets, successor functions, pruning rules etc.

Path Quantifier *A*

Now handling *A* is simple

- Require that hues *h* satisfy
  - (H4) if $A\alpha \in a$ [or $\blacktriangle\alpha \in a$] then $\alpha \in a$.
- Require that Colours *C* satisfy
  - (A1) $A\alpha \in a$ iff $A\alpha \in b$ [and $O\alpha \in a$ iff $O\alpha \in b$]
  - (C2) $a \in C$ and $\neg A\alpha \in a$ [or $\neg\blacktriangle\alpha \in a$] $\implies$ $b \in C$ st. $\neg\alpha \in b$

"$A\alpha$ is in a hue of *C* iff α is in all hues of the same colour"

Path Quantifier ▲: Like *A*?

We would like to require that

- "▲α is in a hue *h* of *C* iff α in all hues that deviate from *h*".

What does it mean for a hue to deviate?

- Hues represent an infinite number of paths
  - Some may deviate. Some may not.
  - Discards too much information

Track deviations in hues?

- *A*Δ: All paths have a deviation.
- ∴ A path has a deviation, which has a deviation . . .
  - Have not found a finite representation.

Path Quantifier ▲

Handle ▲ in hues rather than colours

- If $\blacktriangle N\phi$ then
  - $N\phi$ holds on all 0/now-deviations: $ANO\phi$ in hue.
  - $\blacktriangle\phi$ holds on all possible successors to this hue.

In general: (R5) $\blacktriangle\alpha \in a$ implies $N_a^{-1}(\blacktriangle\alpha)$ in temporal successor b,
Where N_a^{-1} is a formula translation function such that:

$$\left(\bigwedge_{\psi\in a}\psi\right) \rightarrow A\left(\phi \leftrightarrow NN_a^{-1}(\phi)\right)$$

Eventualities

In BCTL* only one type of eventuality: $\alpha U \beta$.
In RoBCTL* an additional eventuality: $\neg\blacktriangle\alpha$
Consider the colour $C = \{\{\neg\blacktriangle Gp, Gp, p, \neg\mathrm{Viol}, \top\}\}$

- C is a valid successor to itself.
- $\neg\blacktriangle Gp \implies$ we must reach $\neg Gp$
  - C alone is not a valid model:
  - If $\neg Gp$ not reachable, must prune C

Note that in BCTL* eventualities don't change

- $(\alpha U \beta)$ remains $(\alpha U \beta)$ until reach β

In RoBCTL* eventualities do change

- E.g. $\neg\blacktriangle N\phi$ becomes $\neg\blacktriangle\phi$ at next step.

BCTL* Complexity

Hues $\approx 2^{|\mathbf{cl}\phi|}$ (powerset of formulae)
Colours $\approx 2^{2^{|\mathbf{cl}\phi|}}$ (powerset of powerset of formulae)
BCTL*: $|\mathbf{cl}\phi| \approx |\phi|$ "doubly exponential"
* Doubly exponential best possible (like CTL*)?

RoBCTL* Complexity

Again # Colours $\approx 2^{2^{|\mathbf{cl}\phi|}}$

$|\mathbf{cl}\phi|$ not elementary, But:

- $|\mathbf{cl}\phi| \in \mathcal{O}(|\phi|)$ if $\blacktriangle \notin \phi$
- $|\mathbf{cl}\phi| \in \mathcal{O}\left(2^{|\phi|}\right)$ if $\blacktriangle \ldots U \notin \phi$
- $|\mathbf{cl}\blacktriangle^n\phi| \in \mathcal{O}\left(n + 2^{2^{|\mathbf{cl}\phi|}}\right)$
- $|\mathbf{cl}\phi| \approx 2\,|\phi|$ for examples (Same as BCTL*).

Informally: Extra 2-exponential blowup only on $\blacktriangle$, U alternations ($\blacktriangle^n$ OK).

- Only if not broken by an A or O

Thus most interesting classes of RoBCTL* formulae are elementary.

RoCTL

- Double exponential complexity too hard for many purposes.
  - CTL* itself is double exponential.
- CTL: EXPTIME
  - Find CTL like fragment of RoCTL.

Expressivity

Is RoCTL* more expressive than CTL*?
We can embed in QCTL* (CTL* + Bisimulation quantifiers $\exists_x \forall_x$)

- QCTL* $\neq$ CTL* (e.g. non-elementary)

Can translate $O\phi$ into CTL*:

$$A(GN\neg\text{Viol} \rightarrow \phi)$$

Can translate some $\blacktriangle\phi$, e.g $\blacktriangle Gp$ becomes

$$pW(E(GN\neg\text{Viol} \wedge Gp)).$$

But no full translation found.

MarkCTL*

We will propose and investigate MarkCTL*

- Adds M operator and special variable m.
- Within scope of M operator, m is true iff on current path.
- Can represent $\Delta\phi$ as $ME(mUGN\neg\text{Viol} \wedge \phi)$

Study RoCTL* from point of view of Hybrid logics?

Summary

- Previously: RoCTL* decidable by reduction to QCTL*
  - Novel Robustly operator
  - Decidable: Infeasible
- Presented Tableau for RoBCTL*
  - May be feasible to decide for some purposes?
    - (but not as feasible as RoCTL)
- Future work
  - Applications
  - Expressivity
    - Hybrid logics, ECTL* etc.
  - Axiomatisation

Any Questions?

References

M^{c}Cabe-Dansted, J. C., French, T., and Reynolds, M. A temporal logic of robustness, RoCTL*. Technical report, UWA, 2007. `http://www.csse.uwa.edu.au/~john/papers/RoCTL07.pdf`.

Reynolds, M. A Tableau for Bundled CTL. *J Logic Computation*, 17(1):117–132, 2007.

$$N_a^{-1}(\phi U \psi) = \left(N_a^{-1}(\phi) \wedge (\phi U \psi)\right) \vee N_a^{-1}(\psi)$$

$$N_a^{-1}(\neg\phi) = \neg N_a^{-1}(\phi)$$

$$N_a^{-1}(N\phi) = \phi$$

$$N_a^{-1}(\phi \wedge \psi) = N_a^{-1}(\phi) \wedge N_a^{-1}(\psi)$$

$$N_a^{-1}(p) = \begin{cases} \bot & \text{if } p \notin a \\ \top & \text{if } p \in a \end{cases}$$

$$N_a^{-1}(A\phi) = \begin{cases} \bot & \text{if } A\phi \notin a \\ \top & \text{if } A\phi \in a \end{cases}$$

$$N_a^{-1}(O\phi) = \begin{cases} \bot & \text{if } O\phi \notin a \\ \top & \text{if } O\phi \in a \end{cases}$$

$$N_a^{-1}(\blacktriangle\phi) = \begin{cases} \bot & \text{if } ANO\Xi\left(N_a^{-1}(\phi)\right) \notin a \\ \blacktriangle\Xi\left(N_a^{-1}(\phi)\right) & \text{otherwise} \end{cases}$$

Expressivity

Is RoCTL* more expressive than CTL*?
We can embed in QCTL* (CTL* + Bisimulation quantifiers $\exists_x$ $\forall_x$)

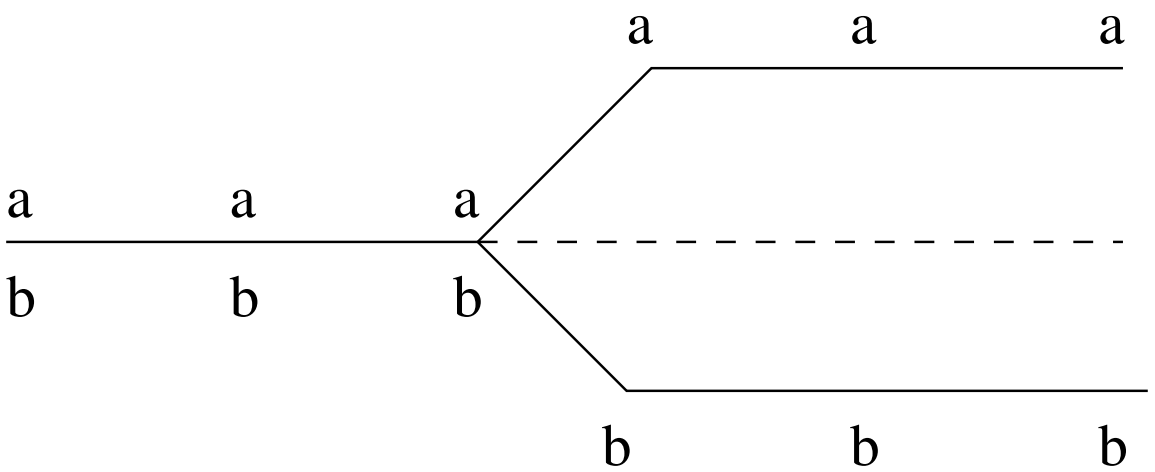
- $\exists_x$: Exists a valuation of variable x
- Satisfiability of $\exists_x \phi$ equivalent to ϕ
- $\neg\exists_x \phi$ not so simple.
- QCTL* $\neq$ CTL*

Expressivity: No $\exists$?

Some formulae do not need $\exists_x$. E.g.: $\blacktriangle Gp$ becomes

$$pW(E(GN\neg\text{Viol} \wedge Gp)).$$

Translation function f from RoCTL* to CTL* exist?
$f(\Delta(Ga \wedge Gb)) \neq f(\Delta Ga) \wedge f(\Delta Gb)$: Ga and Gb must occur on same deviation



Expressivity: About $\wedge$

- Push down $\wedge$: $f(\Delta(Ga \wedge Gb)) = f(\Delta G(a \wedge b))$.
- $f(aUb \wedge cUd)$: break into stages
  - b before d: $a \wedge c$, then $b \wedge c$, then cUd; or
  - b after d; or
  - b and d at same time.
- $f((aUc)Ud)$: circular stages, no finite representation?

CONJECTURE: There exist RoBCTL* formulae that cannot be expressed in BCTL*. Likewise, there exist RoCTL* formulae that cannot be expressed in CTL*.

Maximally Propositionally Consistent

Let *a* be a set of formulae.

Definition

We say that $a \subseteq \mathbf{cl}\phi$ is MPC iff for all $\alpha, \beta \in a$

(M1) if $\beta = \neg\alpha$ then $\beta \in a$ iff $\alpha \notin a$,
(M2) if $\alpha \wedge \beta \in \mathbf{cl}\phi$ then $(\alpha \wedge \beta) \in a \leftrightarrow (\alpha \in a$ and $\beta \in a)$

Hues

A Hue represents is (roughly) a set of formulae that could hold together on some path.

Definition (Hue)

A set $a \subseteq \mathbf{cl}\phi$ is a hue for ϕ iff

(H1) a is MPC;
(H2) if $\alpha U \beta \in a$ then $\alpha \in a$ or $\beta \in a$;
(H3) if $\neg(\alpha U \beta) \in a$ then $\beta \notin a$; and
(H4) if $A\alpha \in a$ or $\blacktriangle\alpha \in a$ then $\alpha \in a$.