Inconsistency–Adaptive Modal Logics: Preliminary Report

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Outline

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Paraconsistent Modal Logics (**pML**)

Are obtained by adding to a paraconsistent logic the modal operators $\Box$ (*necessity*) and $\Diamond$ (*possibility*).

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$\Rightarrow$ too weak to capture actual reasoning!

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Aim

To extend the inconsistency–adaptive framework to modal logics.

$\Rightarrow$ Inconsistency–adaptive modal logics!!

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Paraconsistent Modal Logics

In General

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  = The modal extension of the paraconsistent logic **CL$\bar{\text{u}}$Ns** (equivalent to Priest's **LP**).

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Example

- The logic **TūNs**
  - = The modal extension of the paraconsistent logic **CLūNs** (equivalent to Priest's **LP**).
  - = The paraconsistent counterpart of the modal logic **T**.

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Paraconsistent Modal Logics

The Logic **T$\bar{\text{u}}$Ns**: Language Schema

The Modal Language $\mathcal{L}^{\mathcal{M}}$

language	letters	logical symbols	set of formulas
$\mathcal{L}^{\mathcal{M}}$	$\mathcal{S}$	$\sim, \wedge, \vee, \supset, \equiv, \Box, \Diamond$	$\mathcal{W}^{\mathcal{M}}$

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Definition

- $A \vee B =_{df} \sim(\sim A \wedge \sim B)$
- $A \supset B =_{df} \sim A \vee B$
- $A \equiv B =_{df} (A \supset B) \wedge (B \supset A)$
- $\Diamond A =_{df} \sim\Box\sim A$

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Definition

$\mathcal{S}^{\sim} = \{\sim A \mid A \in \mathcal{S}\}$

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Tu̅Ns–models

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A **Tū̄Ns**–model M is a 4–tuple $< W, w_0, R, v >$, with

- W a set of worlds,
- w_0 the actual world,
- R a reflexive accessibility relation, and
- v an assignment function.

The Assignment Function

AP1 $v\colon \mathcal{S} \mapsto \{0,1\}$.
AP2 $v\colon \mathcal{S}^{\sim} \mapsto \{0,1\}$.

Paraconsistent Modal Logics

The Logic **T$\bar{\text{u}}$Ns**: Semantics

The Assignment Function

AP1 $v\colon \mathcal{S} \mapsto \{0,1\}$.
AP2 $v\colon \mathcal{S}^{\sim} \mapsto \{0,1\}$.

The Valuation Functions

SP1 $v_M\colon \mathcal{W}^{\mathcal{M}} \mapsto \{0,1\}$.
SP2 For $A \in \mathcal{S}$: $v_M(A) = 1$ iff $v(A) = 1$.
SP3 For $A \in \mathcal{S}$: $v_M(\sim A) = 1$ iff $v_M(A) = 0$ or $v(\sim A) = 1$.
SP4 $v_M(\sim\sim A) = 1$ iff $v_M(A) = 1$.
SP5 $v_M(A \wedge B) = 1$ iff $v_M(A) = 1$ and $v_M(B) = 1$.
SP6 $v_M(\sim(A \wedge B)) = 1$ iff $v_M(\sim A) = 1$ or $v_M(\sim B) = 1$.
SP7 $v_M(\Box A, w) = 1$ iff $\forall w' \in W$, if Rww' then $v_M(A, w') = 1$.
SP8 $v_M(\sim\Box A, w) = 1$ iff $\exists w' \in W$ such that Rww' and $v_M(\sim A, w') = 1$.

Truth in a Model

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Models of a Premise Set

The **TūNs**–model M is a model of the premise set Γ iff for all $B \in \Gamma$, it is the case that $v_M(B, w_0) = 1$.

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Semantic Consequence

$\Gamma \vDash_{\mathbf{T\bar{u}Ns}} A$ iff A is true in all **TūNs**–models of the premise set Γ.

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Main Idea

iAML interpret a premise set as consistent as possible.

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$\Rightarrow$ Inconsistencies are supposed to be false, unless or until proven otherwise.

iAL if $A \vee (B \wedge \sim B)$ is derivable then A is supposed to be derivable as well, unless or until it can be proven that $B \wedge \sim B$ might be true.

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Problem

What does it mean to interpret a premise set as consistent as possible in a modal setting?

$\Rightarrow$ As much reachable inconsistencies as possible are supposed to be false!

Inconsistency–Adaptive Modal Logics (**iAML**)

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Reachable Inconsistencies

Inconsistencies that are true in worlds that are reachable from the actual world w_0.

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Inconsistency–Adaptive Modal Logics

General Characterization

Adaptive Logics (**AL**)

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1. A Lower Limit Logic (**LLL**)
   - The **LLL** determines the inference rules that can be applied unrestrictedly.
   - All **LLL**–consequences are **AL**–consequences as well.
2. A Set of Abnormalities Ω
   - Elements of Ω are interpreted as false as much as possible
   - The result: some conditionally derived consequences
     - $\dfrac{A \vee B^{\in \Omega}}{A}$, unless B cannot be considered as false.
3. An Adaptive Strategy
   - The strategy determines which of the conditionally derived formulas have to be withdrawn.

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Example: the Logic $\mathbf{iAT\bar{u}Ns^n}$

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2. The Set of Abnormalities Ω
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Example: the Logic **iAT$\bar{\text{u}}$Ns**$^{\mathbf{n}}$

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   = the paraconsistent modal logic **T$\bar{\text{u}}$Ns**.
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Generalizing the Approach

For all other **iAML**, the characterization is completely equivalent!

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Preferential Semantics

The **iATūNs^n**–consequences of a premise set Γ are defined by reference to sets of preferred **TūNs**–models of Γ.

= the selected sets of **TūNs**–models of Γ.

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Definition

$\Gamma \vDash_{\mathbf{iAT\bar{u}Ns^n}} A$ iff there is at least one selected set Σ of **TūNs**–models of Γ such that A is true in all models that are in Σ.

The Abnormal Part of a Model

$Ab(M) = \{A \in \Omega \mid v_M(A, w_0) = 1\}$

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Minimally Abnormal Models

A **TūNs**–model M of Γ is minimally abnormal iff there is no **TūNs**–model M' of Γ such that $Ab(M') \subset Ab(M)$.

Inconsistency–Adaptive Modal Logics

The Logic **iATūNs**$^{\mathbf{n}}$: Semantics — Selecting the Models

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The Selected Sets

All minimally abnormal models that verify the same abnormalities are grouped together in distinct sets, the selected sets of a premise set.

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All minimally abnormal models that verify the same abnormalities are grouped together in distinct sets, the selected sets of a premise set.

Intuitive meaning: Each selected set Σ captures a minimally abnormal interpretation of the premise set (= a maximally consistent interpretation).

About Consistency

Only those **TūNs**–models of a premise set are taken into consideration that verify as less reachable inconsistencies as possible.

$\Rightarrow$ In case Γ is consistent, the minimally abnormal models will be those models that do not verify any reachable inconsistencies.

= the **T**–models of Γ!!

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⇒ The logic **T** is the Upper Limit Logic of the logic **iATūNs^n**.

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The Logic $\mathbf{iAT\bar{u}Ns^n}$: Example

Theorem

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- $\Gamma \vDash_{\mathbf{T\bar{u}Ns}} A \vee \bigvee(\Delta)$, *and*
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- *$\Gamma \nvDash_{\mathbf{T\bar{u}Ns}} \bigvee(\Delta)$.*

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$\Gamma = \{\Diamond(p \vee (q \wedge {\sim}q)), \Diamond\Diamond(q \wedge {\sim}q), \Box(r \wedge {\sim}r)\}$

$\Rightarrow$ $\Gamma \vDash_{\mathbf{iAT\bar{u}Ns^n}} \Diamond p$, because

- $\Gamma \vDash_{\mathbf{T\bar{u}Ns}} \Diamond p \vee \Diamond(q \wedge {\sim}q)$, and
- $\Gamma \nvDash_{\mathbf{T\bar{u}Ns}} \Diamond(q \wedge {\sim}q)$.

$\Rightarrow$ $\Gamma \nvDash_{\mathbf{iAT\bar{u}Ns^n}} \Box p$, because although

- $\Gamma \vDash_{\mathbf{T\bar{u}Ns}} \Box p \vee \Diamond(r \wedge {\sim}r)$, also
- $\Gamma \vDash_{\mathbf{T\bar{u}Ns}} \Diamond(r \wedge {\sim}r)$.

Theorem

$\Gamma \vDash_{\mathbf{iAS4\bar{u}Ns^n}} A$ iff there is a finite $\Delta \subset \Omega$ such that

- *$\Gamma \vDash_{\mathbf{S4\bar{u}Ns}} A \vee \bigvee(\Delta)$, and*
- *$\Gamma \nvDash_{\mathbf{S4\bar{u}Ns}} \bigvee(\Delta)$.*

Inconsistency–Adaptive Modal Logics

The Logic $\mathbf{iAS4\bar{u}Ns^n}$: Example

Theorem

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- $\Gamma \vDash_{\mathbf{S4\bar{u}Ns}} A \vee \bigvee(\Delta)$, *and*
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R is transitive!

$\Rightarrow$ $\Gamma \nvDash_{\mathbf{iAS4\bar{u}Ns^n}} \Box p$, because although

- $\Gamma \vDash_{\mathbf{S4\bar{u}Ns}} \Box p \vee \Diamond(r \wedge {\sim}r)$, also
- $\Gamma \vDash_{\mathbf{S4\bar{u}Ns}} \Diamond(r \wedge {\sim}r)$.

Theorem

$\Gamma \vDash_{\mathbf{iAK\bar{u}Ns^n}} A$ *iff there is a finite* $\Delta \subset \Omega$ *such that*

- $\Gamma \vDash_{\mathbf{K\bar{u}Ns}} A \vee \bigvee(\Delta)$, *and*
- $\Gamma \nvDash_{\mathbf{K\bar{u}Ns}} \bigvee(\Delta)$.

Theorem

$\Gamma \vDash_{\mathbf{iAK\bar{u}Ns^n}} A$ iff there is a finite $\Delta \subset \Omega$ such that

- *$\Gamma \vDash_{\mathbf{K\bar{u}Ns}} A \vee \bigvee(\Delta)$, and*
- *$\Gamma \nvDash_{\mathbf{K\bar{u}Ns}} \bigvee(\Delta)$.*

Example

$\Gamma = \{\Box(r \wedge {\sim} r)\}$

$\Rightarrow$ $\Gamma \vDash_{\mathbf{iAK\bar{u}Ns^n}} \Box p$, because

- $\Gamma \vDash_{\mathbf{K\bar{u}Ns}} \Box p \vee \Diamond(r \wedge {\sim} r)$, and
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R is not reflexive!

Inconsistency–Adaptive Modal Logics

The Logic $\mathbf{iAK\bar{u}Ns^n}$: Example

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$\Gamma = \{\Box(r \wedge \sim r)\}$

$\Rightarrow$ $\Gamma \vDash_{\mathbf{iAK\bar{u}Ns^n}} \Box p$, because

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R is not reflexive!

$\Rightarrow$ Pseudo–explosion!!

Final Remarks

Conclusion

The inconsistency–adaptive framework can be extended to modal logics in a fairly natural way!

Final Remarks

Conclusion

The inconsistency–adaptive framework can be extended to modal logics in a fairly natural way!

Further Results/Research

- To construct **iAML** based on paraconsistent modal logics that do not contain de Morgan laws (nor their modal analogues).
- To construct **iAML** by starting from the semantic perspective.
- To construct **iAML** that are based on paraconsistent modal logics with a non–reflexive accessibility relation and that avoid pseudo–explosion.

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